NPS-OC-91-005

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ACCURACY OF SHIPBORNE KINEMATIC GPS SURVEYING

by

Barry Grinker

September 1991

Thesis Advisor J.R. Clynch

Approved for public release; distribution is unlimited.

Prepared for:
Naval Postgraduate School
Monterey, California 93943-5000

T260859

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral R. W. West, Jr.
Superintendent

H. Shull
Provost

This thesis is prepared in conjunction with research sponsored and funded by the Naval Postgraduate School (NPS).

Reproduction of all or part of this report is authorized.

Unclassified

curity classification of this page

REPORT DOCUMENTATION PAGE

a Report Security Classification Unclassified		1b Restrictive Markings	
b Security Classification Authority		3 Distribution Availability of Report	
b Declassification Downgrading Schedule		Approved for public release; distribution is unlimited.	
Performing Organization Report Number(s)		5 Monitoring Organization Report Number(s)	
a Name of Performing Organization Naval Postgraduate School	6b Office Symbol *(if applicable)* 35	7a Name of Monitoring Organization Naval Postgraduate School	
c Address *(city, state, and ZIP code)* Monterey, CA 93943-5000		7b Address *(city, state, and ZIP code)* Monterey, CA 93943-5000	
a Name of Funding Sponsoring Organization	8b Office Symbol *(if applicable)*	9 Procurement Instrument Identification Number OM&N	
c Address *(city, state, and ZIP code)*		10 Source of Funding Numbers	
		Program Element No / Project No / Task No / Work Unit Accession No	

1 Title *(include security classification)* ACCURACY OF SHIPBORNE KINEMATIC GPS SURVEYING

2 Personal Author(s) Barry Grinker

3a Type of Report	13b Time Covered	14 Date of Report *(year, month, day)*	15 Page Count
Master's Thesis	From To	September 1991	119

6 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes			18 Subject Terms *(continue on reverse if necessary and identify by block number)*
ield	Group	Subgroup	GPS, Kinematic

9 Abstract *(continue on reverse if necessary and identify by block number)*

In December 1990 an experiment was conducted at the Naval Postgraduate School in Monterey, California in which four ifferent receivers, mounted on the mast of a ship, collected data simultaneously for several hours at a time. Ashtech LD - XII, Trimble 4000 ST, TI 4100 and Magnavox MX 4200 receivers were used.

The reference system consisted of a Krupp Atlas Polarfix laser system set up on the shore at a pre-surveyed site. A two-xis vertical gyro system and a heading gyro gave the ship's 3-dimensional orientation at any instant in time, providing a onnection between the laser reflector and the GPS antennas on the ship's mast. This enabled the reduction of the laser reflector's trajectory to the Ashtech and Trimble antennas for subsequent comparison to the kinematic GPS trajectories of these eceivers determined by the postprocessing of the data collected.

Each data set was processed once with the software provided by the manufacturer and once with an independent software ackage, OMNI, developed by the National Geodetic Survey. In addition to the software, six factors were examined to de-ermine their effects on kinematic GPS surveys. They included : tropospheric corrections, initialization, satellite geometry, phemeris type, data interval and multipath.

In general the software available is versatile; however each package has some limitations which call for further develop-nent. The results indicate that the effects mentioned are small, generally less than the noise in the reference system. An im-ortant factor is the initialization of the kinematic process. Bad initialization can cause a reasonable solution for a period of ime, but a sudden deterioration when the satellite configuration changes. Accuracy levels of a few decimeters were easily chieved with the systems and procedures used. Both Ashtech and Trimble produced trajectories which were accurate to within the noise level of the laser trajectories. In both cases the solution produced by OMNI differed from the solution roduced by the manufacturer's software, only by a few centimeters.

0 Distribution/Availability of Abstract	21 Abstract Security Classification	
☒ unclassified unlimited ☐ same as report ☐ DTIC users	Unclassified	
2a Name of Responsible Individual .R. Clynch	22b Telephone *(include Area code)* (408) 646 - 3721	22c Office Symbol OC/CL

D FORM 1473,84 MAR 83 APR edition may be used until exhausted
All other editions are obsolete

security classification of this page

Unclassified

Approved for public release; distribution is unlimited.

ACCURACY OF SHIPBORNE KINEMATIC GPS SURVEYING

by

Barry Grinker
Lieutenant Commander, Israeli Navy
B.S., Hebrew University - Jerusalem, 1979

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN HYDROGRAPHIC SCIENCE

from the

NAVAL POSTGRADUATE SCHOOL
September 1991

ABSTRACT

In December 1990 an experiment was conducted at the Naval Postgraduate School in Monterey, California in which four different receivers, mounted on the mast of a ship, collected data simultaneously for several hours at a time. Ashtech LD - XII, Trimble 4000 ST, TI 4100 and Magnavox MX 4200 receivers were used.

The reference system consisted of a Krupp Atlas Polarfix laser system set up on the shore at a pre-surveyed site. A two-axis vertical gyro system and a heading gyro gave the ship's 3-dimensional orientation at any instant in time, providing a connection between the laser reflector and the GPS antennas on the ship's mast. This enabled the reduction of the laser reflector's trajectory to the Ashtech and Trimble antennas for subsequent comparison to the kinematic GPS trajectories of these receivers determined by the postprocessing of the data collected.

Each data set was processed once with the software provided by the manufacturer and once with an independent software package, OMNI, developed by the National Geodetic Survey. In addition to the software, six factors were examined to determine their effects on kinematic GPS surveys. They included : tropospheric corrections, initialization, satellite geometry, ephemeris type, data interval and multipath.

In general the software available is versatile; however each package has some limitations which call for further development. The results indicate that the effects mentioned are small, generally less than the noise in the reference system. An important factor is the initialization of the kinematic process. Bad initialization can cause a reasonable solution for a period of time, but a sudden deterioration when the satellite configuration changes. Accuracy levels of a few decimeters were easily achieved with the systems and procedures used. Both Ashtech and Trimble produced trajectories which were accurate to within the noise level of the laser trajectories. In both cases the solution produced by OMNI differed from the solution produced by the manufacturer's software, only by a few centimeters.

TABLE OF CONTENTS

I. INTRODUCTION 1
A. GENERAL. 1
B. THESIS DESCRIPTION AND OBJECTIVES. 1
C. SATELLITE RADIO POSITIONING. 2
D. SATELLITE POSITIONING SYSTEMS. 3
1. Transit 3
2. The GLOBAL POSITIONING SYSTEM (GPS) 4
3. General Observation Equation. 6
4. Kinematic GPS. 6
5. GPS Measurements. 7

II. BACKGROUND 9
A. GENERAL 9
1. Satellite Orbits 9
a. Broadcast Ephemerises. 11
b. Precise Ephemerises. 12
2. The GPS Signal 14
3. Measurements. 15
a. Pseudorange Measurements. 15
b. Carrier Phase Doppler Measurements. 16
B. PROPAGATION EFFECTS 17
a. Ionospheric Dispersion Effects 17
b. Tropospheric Effects 18
C. OBSERVATION EQUATIONS 19
1. Pseudo Range Observable 19
2. Continuous Carrier Phase Observable 20
3. Integrated Doppler 21
D. DIFFERENCING TECHNIQUES 23
1. Single Differences 23
2. Double Differences 25
3. Triple Differences 27

E. GPS SATELLITE GEOMETRY AND ACCURACY. 28
F. MULTIPATH 30
G. SELECTIVE AVAILABILITY - SA. 30

III. KINEMATIC GPS 32
A. CONCEPTS 32
1. Kinematic Positioning Models. 32
B. AMBIGUITY RESOLUTION. 33
1. Initialization Techniques 34
a. Static Survey Initialization. 34
b. Antenna Swap. 34
c. External Baseline Input. 36
2. Cycle Slips. 36
C. SOLVING FOR POSITION. 37

IV. THE MONTEREY BAY PRECISE POSITIONING EXPERIMENT.
(MBPPE) 40
A. GENERAL. 40
B. OBJECTIVES OF THE EXPERIMENT 42
C. EQUIPMENT 42
D. METHOD 42
1. Planning and preparation 43
2. In Port Reference Data 46
3. At Sea Data 47
E. GPS DATA. 50

V. DATA PROCESSING PROCEDURES 51
A. GENERAL 51
B. ASHTECH DATA PROCESSING. 51
1. Method 51
2. Day 340 processing. 53
3. Day 341 processing. 54
4. Using Precise Ephemerises. 54
5. Results of Ashtech processing. 54
C. TRIMBLE DATA PROCESSING. 55

measurement interval (1 second for example), enables the user to position the platform on which the receiver is mounted. This method requires the second receiver set up at a fixed, " known " station, simultaneously collecting data. For " real time " navigational uses, a radiolink between the fixed and " rover " (moving) receivers passes data from the receiver at the known site to the unknown site. For surveying purposes a radiolink is not necessary and the measurements are incorporated into the post-processing programs. Each point in the " rover " receiver's trajectory is considered as a new baseline to the fixed receiver. This method is known as " Differential GPS " and can be used both for static and kinematic surveys.

5. GPS Measurements.

In GPS surveying there are essentially two methods of measuring the distance between a satellite and a receiver's antenna.

1. The Range measurement
2. The Phase measurement

The Range measurement determines the time lapse between the instant a signal is transmitted from the satellite and the instant the signal arrives at the antenna. This time difference is then multiplied by the assumed speed of light to get the distance by,

$$\rho_R = \Delta t\, c$$

Any error in the measurement of the transmission time, the received signal time or in the speed of light (propagation effects) will automatically create an error in the range measurement between satellite and antenna and subsequently in the receiver's position.

Measuring the phase of a signal involves three separate issues. Once the signal is received it is possible to measure the instantaneous phase, the fraction of a cycle at a given time. As long as the signal is continuously tracked, it is easy to count the number of full cycles received since the initial receive time. The number of full cycles the signal went through, between the transmitter and the receiver, prior to the initial measurement in the receiver, is a factor which has to be determined. This is termed the cycle ambiguity.

The phase measurement compares the phase of the signal as it left the satellite and when it arrived at the antenna. Once we know the number of full cycles the carrier wave has gone through in its path, the phase measurement determines the range between satellite and antenna to sub-cycle accuracy. The wavelength for the L_1 carrier wave is about 19 cm long, enabling the receiver to easily determine the range to centimeter level

(typically less than half a wavelength error or 180 degrees in the phase measurement). As long as the cycle count is determined accurately at an initial instant in time and lock is not lost with the satellite, this measurement gives the best results for positioning the receiver's antenna. The distance in this case is determined by the basic equation :

$$\rho_{\phi} = N\lambda + \phi\lambda$$

where N is the integer number of wavelengths and ϕ is the fractional part of the wavelength (phase) measured in cycles.

In practice neither of these methods is used directly. This is only a basic, conceptual introduction to the scientific approach to surveying using a satellite radio positioning system. The next chapter presents a detailed description of the mathematical models involved in GPS and the development of the techniques used.

II. BACKGROUND

A. GENERAL

This chapter explains and elaborates on the terms and concepts used in GPS. Many books have been written on the subject of GPS surveying. This chapter however is intended only to present a concise overview for the reader who is not familiar with GPS, emphasizing the kinematic mode of surveying and all that pertains to it. The next chapter describes the differential kinematic procedure in detail.

1. Satellite Orbits

As mentioned in the introduction, one of the essential features of GPS is the determination of the satellite positions. Measuring the range between a receiver whose position is unknown and a transmitter whose position is also unknown is pointless. For differential, relative positioning however, a high degree of accuracy in the satellites' instantaneous coordinates is not necessary. Considering the distance between the satellites and the receivers compared to the distance between two receivers in differential mode surveying, errors in the satellite position will be almost entirely eliminated in the differencing process (described further on in this chapter). What is of greater importance is the time tag on the satellite positions given. With a speed of almost 4 km/s, the satellite positions can easily deteriorate if a time discrepancy is introduced into the system at any stage. With a number of time frames in GPS --- satellite time, receiver time, GPS time and UTC --- it is imperative that time-tagging be well defined and cautiously implemented.

A few words on orbital elements are necessary to enhance the understanding of the satellite motion. Six forces contribute to the satellite motion :

1. Gravitational attraction of the earth -- the greatest force.
2. Gravitational attraction of the sun, moon and planets -- weaker forces due to the distances involved (called third body effects).
3. Atmospheric drag force -- very small at 20,000 km above the earth's surface.
4. Solar radiation pressure.
5. Magnetic forces.
6. The variable part of the earth's gravitational field arising from tidal and other deformations of the solid earth and the oceans.

The main characteristics of the satellite motion are determined by the major force, the earth's gravitational attraction, which is three orders of magnitude larger than all the other forces together. The remaining forces listed vary in time and cause perturbations in the satellite's orbit.

The equations of motion for satellites are differential equations that are solved by numerical integration over time. Integration begins with initial conditions, such as the position and velocity of the satellite at some initial instant (epoch). The basic equation underlying orbital theory is Newton's law of gravitation,

$$F = Gm\frac{M_E}{r^2}$$

where G is the universal gravitational constant, m is the mass of the satellite, M_E is the mass of the earth and r is the geocentric distance to the satellite.

This is actually a simplification. The spherical harmonic expansion to the order 9 by 9 is used to model the earth's gravitational potential. This higher order expansion accounts for the small perturbations in the earth's gravity field and improves accuracy of the orbit prediction. A more detailed development of these spherical harmonics can be found in many books on physical geodesy, such as " Physical Geodesy " by Heiskanen and Moritz [Ref. 3]. Kepler's laws follow from Newton's law of gravitation and they define the orbital motion of the satellites. Six Keplerian elements are needed to position a satellite in space. Three, (Ω, ω, i) position the orbital ellipse in space and the other three, (a,e,f) determine the position of the satellite within the orbital plane. Figure 2 shows the relationships between the six Kepler elements.

The definitions of the Kepler elements are:

1. Ω = right ascension of the ascending node -- the geocentric angle between the nodal directions and vernal equinox measured in the equatorial plane.
2. ω = argument of perigee -- the angle between the nodal and perigee directions measured in the orbital plane.
3. i = inclination -- the angle between the equatorial and orbital planes.
4. a = semi-major axis of the elliptical orbit. (b = semi-minor axis of the elliptical orbit).
5. e = eccentricity of the orbit ($e = \frac{a-b}{a}$).
6. f = an element describing the position of the satellite on the orbital ellipse at a given time; usually one of the anomalies, true, eccentric or mean, which relate satellite position in the orbit to time, is used.

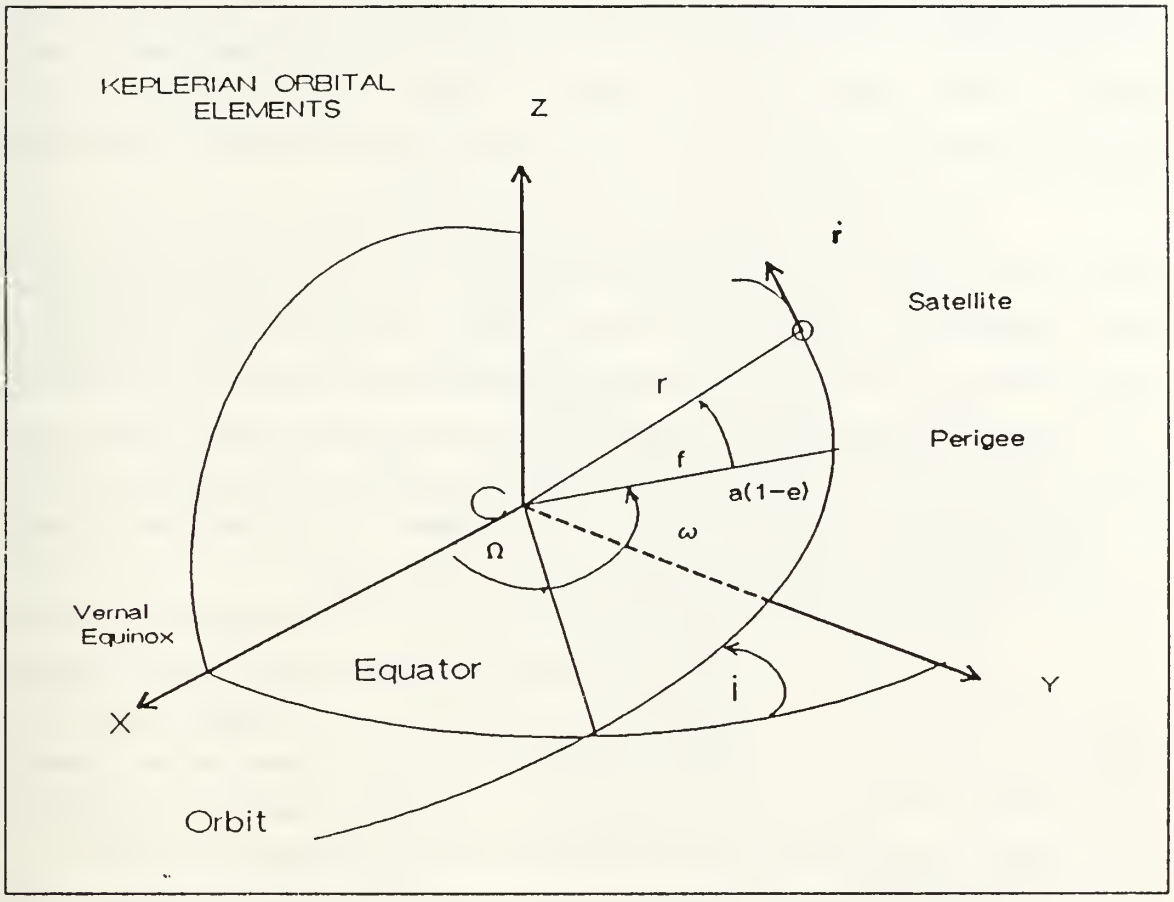

Figure 2. The Orbital Plane: See text for definition of the symbols.

The mean anomaly M is a fictitious angle defined by $M = n(t - t_p)$ where :

$n = (GM_E/a^3)^{1/2}$ is the mean motion,

t = epoch when the satellite position is required

t_p is the time of perigee crossing.

a. Broadcast Ephemerises.

The broadcast ephemeris information transmitted by the satellites is computed and controlled by the Master Control Segment. Five ground stations, at Colorado Springs, Hawaii, Ascencion, Diego Garcia and Kwajalein, are monitor stations with Colorado Springs being the master station. Ephemeris prediction is a two step proce-

dure. First a reference ephemeris is produced from data gathered over a week at the five monitor stations. Predicted ephemerises are then periodically produced based on additional " on-line " pseudorange and integrated Doppler measurements (nominally 15 minutes of smoothed data recorded at 1.5 second intervals) at each of the monitor stations. Figure 3 shows the ephemeris prediction procedure.

The reference ephemeris and the " on-line " predicted ephemerises are compared and discrepancies produced. These discrepancies are corrected for known biases, such as ionospheric and tropospheric effects, satellite and monitor station antenna phase center offsets and relativistic effects resulting from the earth's rotation and satellite motion. The pseudorange measurements are smoothed and output every 15 minutes for each satellite-station pair. The integrated Doppler measurements are sampled every 15 minutes. The data sets thus produced are then processed by a Kalman filter estimator to produce estimates of the satellite positions and velocity states.

The orbital element perturbations, output by the Kalman filter, are used to correct the reference ephemeris, and an extrapolated ephemeris is predicted for upload to the satellite. Once the full constellation of GPS satellites is operational, predicted ephemeris errors of about 1 m radially, 7 m along track and 3 m cross track are expected. The broadcast ephemerises are transmitted in the navigation message of the satellite. Each satellite transmits orbital information for the entire satellite constellation. At a minimum the ephemeris data is uploaded to the satellites daily. The satellites also keep track of the time lapse from the latest ephemeris upload. The orbital parameters transmitted describe the predicted satellite orbit for the period for which they are intended -- about an hour from a determined reference epoch. The processing software uses a polynomial interpolation algorithm to determine the positions of the satellites at the required epochs, between the reference epochs.

b. Precise Ephemerises.

Postcomputed or " precise " ephemerises serve various purposes. For absolute positioning and other high accuracy purposes a better position of the satellites is required. The orbital determination procedure is similar to that of the reference orbits for the broadcast ephemeris. However, in addition to the five monitor stations, stations in Australia, Seychelles, England and Argentina augment the data base used. A more extensive international network is planned for the future.

The precise ephemerises are available on a weekly basis, usually with a few weeks delay. However, since they are used only in post-processing, this is not of importance. The parameters produced in the precise ephemeris file describe the satellite orbits

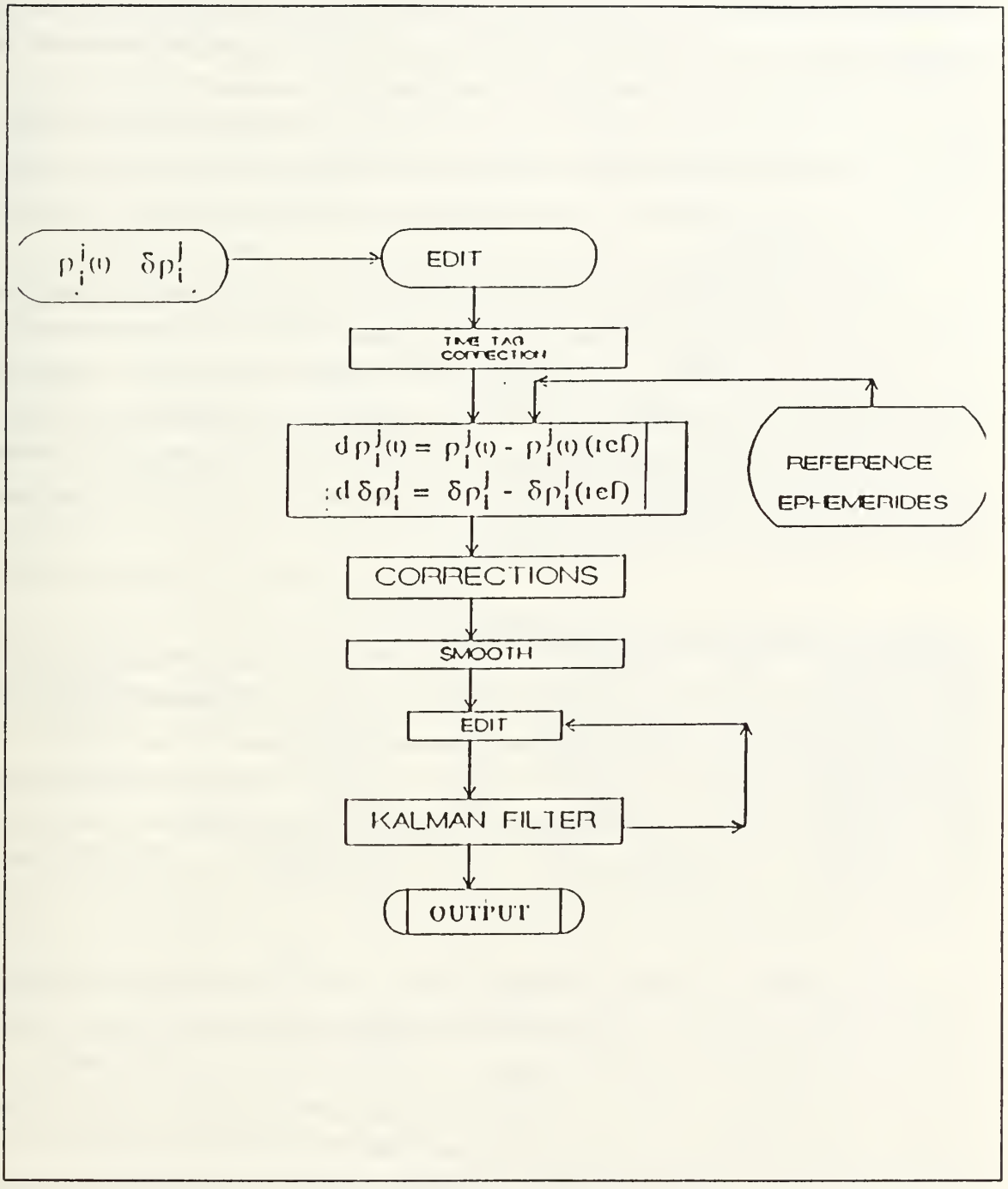

Figure 3. Ephemeris Prediction Procedure.: Here ρ is the range, the subscripts refer to the receivers and the superscripts to the satellites.

at 15 minute intervals. Most of the post-processing programs available today can accommodate both broadcast and precise ephemeris files. The main difference between the broadcast and the precise ephemerises lies in the method used to obtain each of them. The broadcast ephemeris is obtained by extrapolation, predicting the future positions of the satellites based on data that could be up to 36 hours old. The precise ephemerises on the other hand are obtained using interpolation of actual data recorded.

2. The GPS Signal

All GPS transmissions are coherently derived from the fundamental frequency of $f_0 = 10.23 MHz$. The carrier frequencies are multiples of the fundamental frequency - $L_1 = 154 f_0 = 1575.42\ MHz$, $L_2 = 120 f_0 = 1227.60 MHz$ The resulting wavelengths for the two carriers are about 19 cm and 24 cm respectively. The chipping rate of the P - code is equal to the fundamental frequency - 10.23 MHz, while that of the C / A - code is one-tenth of the fundamental frequency - 1.023 MHz. Table 2 shows the important features of the GPS code frequencies [Ref. 4].

Table 2. SUMMARY OF GPS SIGNALS

Feature	C / A	P	Message
Chipping rate	1.023 M bps	10.23 M bps	50 bps
Wavelength	293 m	29.3 m	N / A
Repetition	1 ms	1 week	N / A
Attributes	Easy to acquire	Precise positioning Jam resistant	Time & Ephemeris

The precision P - code is the principal code used by the military in a navigation mode. It is a Pseudo - Random Noise (PRN) code generated by mathematically mixing two other codes. The P - code does not repeat itself for 267 days (over 38 weeks), enabling the assignment of weekly portions of the code to the different satellites. Each satellite therefore, while transmitting on the common carrier frequency, can be identified by the portion of P - code it is using. All codes are initialized at midnight between Saturday and Sunday (GPS time), thus defining the GPS week as an important unit of time. Since there are fewer than 38 GPS satellites in orbit, there are some unused weekly portions of the P - code. Both L_1 and L_2 carriers are modulated by the P - codes.

The coarse / acquisition (C / A) code is only one ms long. The resulting wavelength causes it to be used for lower accuracy, general purposes. The C / A code

is synchronized with the P - code. Each satellite has its own individual code making it possible to easily distinguish one satellite from another. The C / A - code is used mainly on the L_1 carrier; however, as a ground-control option, it can be used on L_2.

The navigation message is modulated on both L_1 and L_2 carriers at a bit rate of 50 bps. It contains information on the ephemerises of the satellites, GPS time, satellite clock behaviour and system status messages.

Relativistic effects are important for precise GPS positioning and need to be accounted for. The general relativity effect is compensated for by a frequency offset. The offset is determined by the equation $\Delta f / f = 4.45 x 10^{-10}$. This results in a Δf of 0.00455 Hz which is implemented in the satellite production stage with the fundamental frequency set at 10.22999999545 MHz. A second general relativity error, caused by the satellite's orbit not being exactly circular, is proportional to the eccentricity of the orbit. It is possible to calculate the effect and apply a range correction to account for this effect. This is usually done in the receiver. However, for relative positioning, this effect will be eliminated in the differencing process.

More details on the GPS signal specifications can be found in the GPS Interface Control Document - I.C.D. 200 [Ref. 5].

3. Measurements.

GPS is a one-way ranging system. This implies that two separate instruments are involved in the system, one transmitting and one receiving a signal. Measuring the time lapse of the signal, from transmission to reception, calls for precise correlation or synchronization of the instruments. In effect, with each individual receiver having its own clock drift, as well as the satellites having their own clock drifts, synchronization is practically impossible. Instead all the satellite clocks are mathematically related to a single time frame - GPS time - with their offsets and drifts constantly monitored by the control stations. The relevant satellite clock corrections are then computed and sent to the satellites in the upload message. The satellites are therefore all mathematically, though not physically, synchronized to GPS time.

The corrections for all satellite clocks are broadcast to the users (receivers) in the GPS navigation message. The problem of synchronizing the receiver clock to the common time frame remains, but this is easily solved.

a. Pseudorange Measurements.

Most receivers developed are code correlating receivers. The receiver generates a duplicate of the signal received, aligns it with the satellite signal using a delay - lock - loop and continues to track it. The aligning of the receiver generated signal with the satellite

signal is achieved by cross-correlation and applying a necessary correction to the receiver clock which generates the signal. This is done until a match is obtained. The code generating clock, duplicating the satellite signal, is now essentially keeping the satellite time. The offset of the code clock from the receiver clock (which is assumed to be keeping GPS time) is the time of flight of the signal, which when multiplied by the speed of light (c), produces the pseudo-range. The offset of the actual receiver clock from GPS time, and the range bias it produces, are parameters solved for in the observation equations (τ). The measured quantity is the time difference between signal transmission and reception.

The precision of the pseudo-range measurements is about 1 % of the period between successive code bits. For the P - code successive bits are about 0.1 microseconds apart ($f_P \simeq 10MHz$) implying a measurement precision of 1 nanosecond [Ref. 6]. When multiplied by the speed of light this results in a range measurement precision of about 30 cm. For the C / A - code, this number is multiplied by a factor of 10, resulting in a measurement precision of about 3 m. In reality these numbers may be larger by a factor of two or three.

b. Carrier Phase Doppler Measurements.

The carrier beat phase is the phase of the signal which remains when the incoming Doppler shifted carrier wave is differenced (beat) with the constant frequency generated by the receiver. This measurement is obtained as a by-product of the correlation channel or from a squaring channel.

Because the wavelength of the carrier is much shorter than the wavelengths of the codes, the precision of the carrier beat phase measurements is much higher than that of the code pseudo- ranges. For the L_1 carrier signal, with a wavelength of 19 cm, the phase measurement made to about 1 % of the wavelength results in a precision of about 2 mm, two orders of magnitude better than the P - code pseudo-range.

The disadvantages of the carrier beat phase measurement relate to the cycle ambiguity problem. Measuring the observed phase within a particular cycle is easy, but determining the initial number of integer cycles from transmission to reception, is more complicated. Maintaining lock, once cycle ambiguity has been resolved and the continuous phase count begun, is essential for kinematic applications. Only under favorable conditions, such as tracking five or more satellites with only one losing lock at any instant, can the processing software overcome this problem.

B. PROPAGATION EFFECTS

GPS signals behave like any other electromagnetic signals. With the transmission source and the receiver in two completely different environments, it is important to understand the effects of the various media through which the signal passes. Were the entire path of the propagating signal in a vacuum, there would be no need for any corrections. However, this is not the case and so the effects of the various media have to be accounted for. The two most important areas affecting the GPS signal in its path are the ionosphere and the troposphere. The effects on the signal have to be either modelled or eliminated by differencing or other techniques.

a. Ionospheric Dispersion Effects

The ionosphere is a region in the atmosphere, between 50 and 1000 km above the surface of the earth, where gas molecules have been ionized by ultra-violet solar radiation. Any electromagnetic signal propagating through an ionized medium is affected by the characteristics of the medium. The change in path length due to the ionosphere depends on two factors : the index of refraction of the medium (a function of time and place) and the path length of the signal through the ionosphere (a function of the elevation angle of the satellite). The index of refraction of the medium depends upon the electron density, N, in electrons/m^3, and the inverse square of the frequency. The electron density varies in time and place. The index of refraction is given by the equation :

$$n_p \simeq 1 - a\,N/f^2 \qquad\qquad n_g \simeq 1 + aN/f^2$$

where n_p is the phase index of refraction, n_g is the group index of refraction, N is the electron density, " a " is a constant and f is the frequency.

GPS code - signals are dependent upon the group index of refraction with the ionospheric group delay given by

$$\Delta\rho_g = \int (n_g - 1)ds = + aN_T/f^2$$

Phase measurements are influenced by the phase index of refraction with the ionospheric phase delay given by

$$\Delta\rho_p = \int (n_p - 1)ds = - aN_T/f^2$$

In both the above cases N_T is the total electron content along the propagation path in electrons/m^2 with typical values of $10^{16} - 10^{18}$ electrons/m^2. For $\Delta\rho$ given in meters, and f given in Hz, $a \simeq 40.28$. It is evident that for pseudo-range and phase measurements the magnitudes of the ionospheric corrections are identical while the signs are opposite. The magnitude of the ionospheric correction for GPS varies from tens of centimeters to tens of meters with the lower value obtained between midnight and early morning when N_T is lowest.

Since the ionospheric effect is frequency dependent, L_1 and L_2 measurements can be compared to estimate the effect. A linear combination of the frequencies results in the ionospheric correction for L_1 given by

$$d_{ion}(L_1) = [\rho(L_1) - \rho(L_2)][f_2^2 / (f_2^2 - f_1^2)]$$

where d_{ion} is one of the corrections applied to the computed range to get the measured range between satellite and receiver.

b. Tropospheric Effects

Refraction in the atmosphere, including the troposphere and the region up to about 80 km in altitude, is frequency independent below about 30 GHz. Therefor the group and phase delays are the same and d_{trop} has the same magnitude and sign for both the pseudo-range and the phase measurements.

The troposphere can be divided into two main components - dry air and water vapor. The " dry " component contributes about 90 % of the total tropospheric refraction. The " wet " component is hard to predict since localized concentrations of water vapor are abundant in the atmosphere. Fortunately the contribution of the wet part is small - only about 10 % of the total. Despite the variability of the wet component attempts have been made to model it. Other than modelling the troposphere based on meteorological measurements (temperature, pressure and relative humidity) at the GPS sites, it is possible to take water vapor measurements along the line of transmission. This however is a costly procedure involving instruments called Water Vapor Radiometers.

The tropospheric range correction to the measurements is on the order of about 1.9 to 2.5 m in the zenith direction and increasing approximately with the cosecant of the elevation angle. The maximum correction is on the order of about 25 m. Differencing the measurements over short baselines greatly reduces the tropospheric effect. However, with a reference station on land and the mobile receiver on a ship, the water vapor content along the two propagation paths could differ greatly. This can easily

introduce errors at the centimeter level. Another factor which can introduce errors is altitude differences between the receivers. In our Monterey Bay experiment both reference and mobile receivers were near sea level, with the same tropospheric thickness above each of them. Were the reference station set up at a higher elevation, this would have introduced a larger difference in tropospheric error into the measurements.

C. OBSERVATION EQUATIONS

GPS is an electronic ranging system whereby positions are obtained by measuring the ranges to certain transmitters. The unknown parameters lie in the observation equations which model the method used. On the left hand side we have the measured quantity, time difference or phase angle, and on the right hand side the assumed factors which make up the measurement. In differential kinematic GPS the observation equations are modified with the measurement being comprised of a linear combination of phase measurements. This is described in detail in the next chapter.

1. Pseudo Range Observable

As mentioned before, in the pseudo- range method the measured quantity is the time of flight of the signal. Considering the GPS time-frame as the one to which all measurements are related, this time lapse can be written as $d\tau = T(\tau_r) - t(\tau_x)$ where T is the time in the receiver time - frame and t is the time in the satellite time-frame. Adding and subtracting the elapsed GPS time $\tau_r - \tau_x$ we get

$$d\tau = (\tau_r - \tau_x) + [\tau_x - t(\tau_x)] - [\tau_r - T(\tau_r)]$$

The first term on the right is ideally the travel time of the signal which, when multiplied by the speed of light, gives us a measure of the true range to the satellite. The second term on the right represents the satellite clock offset from GPS time and the third term represents the receiver clock offset. The true range to the satellite is represented by

$$\rho = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$

However, the satellite position is only assumed to be precisely known. In fact it contains some error which, combined with the random noise in the system, can be considered as $d\rho$. Taking into account the ionospheric and tropospheric effects as well as ephemeris error and random noise, the pseudo-range observation equation is given by :

$$\rho_R = c\,d\tau = \rho + d\rho + c\,(\tau_s - \tau_r) + d_{ion} + d_{trop}$$

We are primarily interested in the receiver coordinates which are found in the true range term, ρ.

2. Continuous Carrier Phase Observable

The carrier signals, L_1 and L_2, are continuous signals transmitted by the satellites. Determining the range to a satellite is a multi-step process. First the satellite has to be identified since all satellites transmit the same carrier frequencies. This is accomplished by observing the P - code or C / A - code. Once the satellite has been identified, the receiver must align its self-generated signal to the incoming satellite signal. This is readily accomplished in a squaring or code correlation channel. Once locked on to the satellite signal, the receiver can easily follow the phase and measure both phase angle and integer number of full cycles since tracking began. However, the problem of determining the integer number of cycles between the satellite at transmission and the receiver at reception remains. This is commonly refered to as the cycle ambiguity.

The total phase observed, ϕ_{total}, consists of a measured fractional part, $Fr(\phi)$, an integer cycle count, $Int(\phi)$, and the unknown cycle ambiguity, N. This can be written as $\phi_{total} = Fr(\phi) + Int(\phi,t_0,t) + N(t_0)$. As long as lock is maintained and no cycle - slips occur, the first two terms on the right hand side are readily measured. The integer ambiguity remains a parameter that needs to be solved for. Figure 4 illustrates the carrier phase observable with all its components. The unknown integer phase value at reception has to be determined. Once the receiver locks onto the signal, a cycle count, of whole cycles and an instantaneous fraction of a cycle, can be readily measured.

In the case of the pseudo-range measurement, the time measured was readily converted to a measure of distance by multiplying by the speed of light. In the phase measurement, distance is obtained by multiplying the phase count by the signal wavelength. Since $\lambda = \frac{c}{f}$ the total phase, in cycles, can be written as

$$\phi_{total} = -(f/c)\rho - f(\tau_s - \tau_r) - (f/c)(-d_{ion} + d_{trop}) + N \quad cycles$$

where f is the frequency, c is the speed of light and the rest of the terms equivalent to the pseudo-range case. The f / c term is the inverse of the wavelength which, when multiplied by a range results in a phase measurement in cycles. The phase-range observable is now given by :

$$\rho_\phi = -\lambda\,\phi_{measured} = \rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + noise.$$

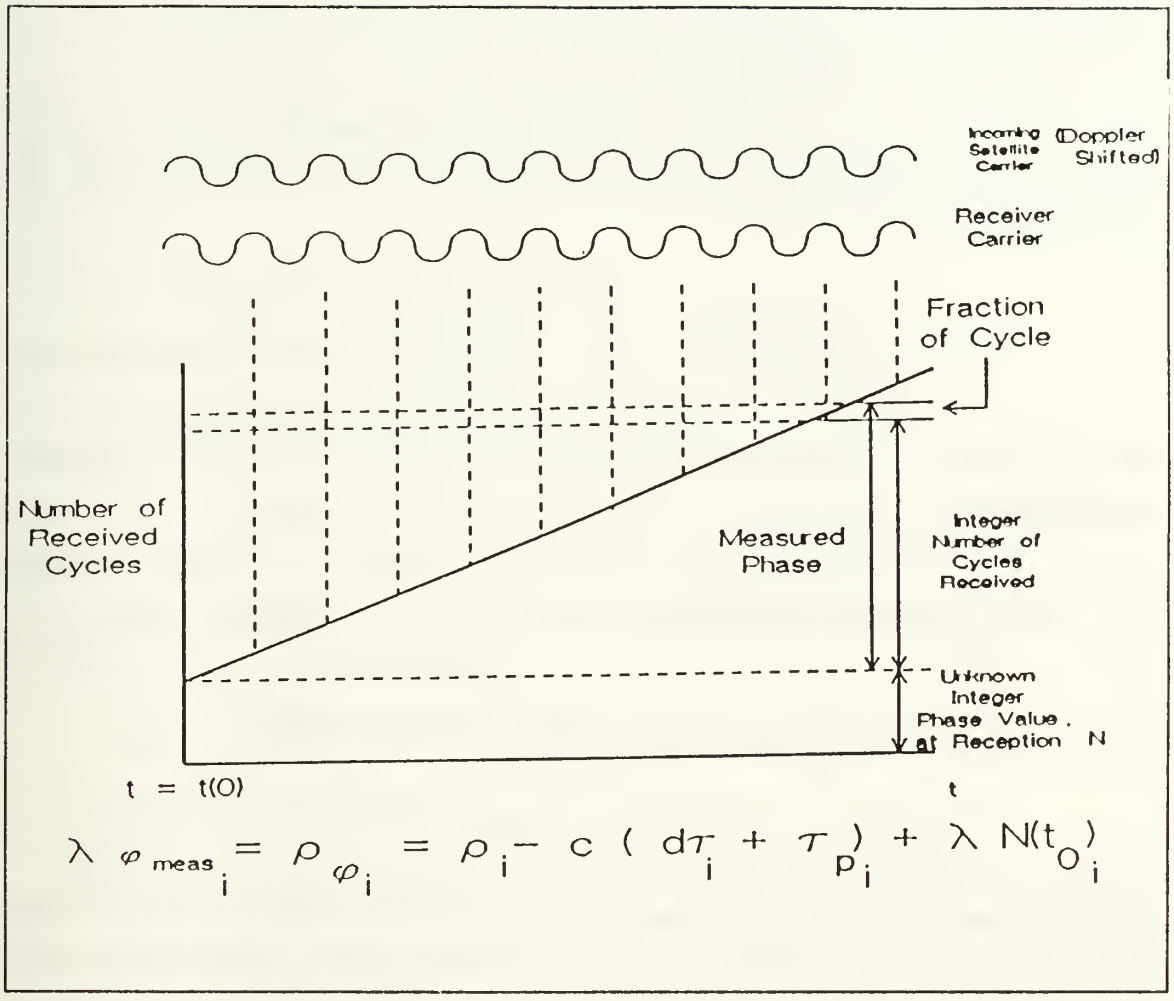

Figure 4. Continuous Carrier Phase.

As in the comparible pseudo-range equation, the desired receiver coordinates are found in the true range ρ. The integer ambiguity term has been added and the sign of the ionospheric effect has changed.

3. Integrated Doppler

Until now we have regarded the transmitter and receiver as being stationary with regard to each other. In effect the transmitter (satellite) is moving relative to the receiver, whether the receiver is fixed or mobile. This causes the Doppler effect to change the received frequency from the satellite transmitted frequency. This frequency change is dependent upon the relative geometry, and therefore positions, of the receiver and satellite. The Doppler frequency is given by the expression.:

$$f_{Dop} = -(f_x \frac{v}{c}) = -f_x \frac{\dot{R}}{c}$$

where f_x is the transmitted frequency, c is the propagation speed and $v = R$ is the radial velocity of the satellite. Integrating the above expression over time results in the following expression :

$$\int_{t_1}^{t_2} f_{dop} = \frac{-f_x}{c} \int_{t_1}^{t_2} \dot{R}(t)$$

Since the frequency is in fact the first derivative of the phase, the above expression can be rewritten as :

$$\int_{t_1}^{t_2} \dot{\phi}(t) = \frac{-f_x}{c} \int_{t_1}^{t_2} \dot{R}(t)$$

Carrying out the integration over two discrete time epochs results in :

$$\phi(t_2) - \phi(t_1) = \frac{-f_x}{c} [R(t_2) - R(t_1)]$$

where R is the radial distance between the receiver and the satellite and ϕ is the total phase angle of the signal. This results in the " phase residual " given by the expression

$$\Delta\phi = \frac{-f_x}{c} \Delta R$$

which gives us a measure of the change in relative position between satellite and receiver.

The carrier beat phase observable is obtained when the incoming Doppler shifted signal is beat with the nominally constant reference frequency generated by the receiver. From the phase / frequency relationship, which is valid for stable oscillators over short time intervals, we get the following expression :

$$\dot{\phi} = \frac{\phi(t + \delta t) - \phi(t)}{\delta t} = f$$

Letting $\delta t = T - t$ we get

$$\phi_\alpha(T) = \phi^k(t) + f\,(T - t) \quad cycles$$

where the subscript α refers to a receiver and the superscript k refers to a satellite. The measured phase is the difference between the phase of the signal that left the transmitter and the phase of the receiver generated signal.

This results in

$$\phi = \phi^k(t) - \phi_\alpha(T) = -f\,(T - t)$$

The fundamental approximation relating transmit and receive times is :

$$t + \tau_s + (\rho - d_{ion} + d_{trop} + noise)/c = T + \tau_r$$

where $t + \tau_s$ is the satellite time with offset, $T + \tau_r$ is the receiver time with receiver clock offset and the term in parentheses, divided by the speed of light, is the time of flight of the signal. Rearranging to get T - t and substituting from the previous equation, results in the carrier beat phase model :

$$\phi = -(f/c)\rho - f\,(\tau_s - \tau_r) - (f/c)\,(-d_{ion} + d_{trop}) + noise \qquad cycles$$

D. DIFFERENCING TECHNIQUES

Differencing refers to the technique whereby we eliminate or isolate a certain parameter by forming linear combinations of the observables. Of the error sources affecting the GPS signals, satellite orbit, satellite clock, receiver clock and atmospheric propagation errors show some correlation over short baselines. Even for two receivers 100 km apart, a satellite signal transmitted from 20,000 km away passes through almost identical media. Both the ionosphere and the troposphere have very small lateral changes in their properties and the path length of the signal to each of the receivers hardly differs. Three levels of differencing, according to the number of linear combinations, can be made.

In the following discussion a standard notation will be used to distinguish between the various measurements. Subscripts α, β etc. refer to receivers; superscripts i, j, k etc. refer to satellites; ϕ refers to a measure of phase while ρ_ϕ refers to a range based on the phase measurement; τ is the clock offset of either the satellite or the receiver.

1. Single Differences

There are three kinds of single differences that one can take.

1. Single differences over receivers - where there are two receivers observing the same satellite simultaneously.

2. Single differences over satellites - where one receiver observes two satellites at a given instant in time.
3. Single differences over time - where one receiver observes one satellite at two sequential instants in time.

In each of these cases different errors are " differenced " out. Figure 5 shows the between - receiver single difference.

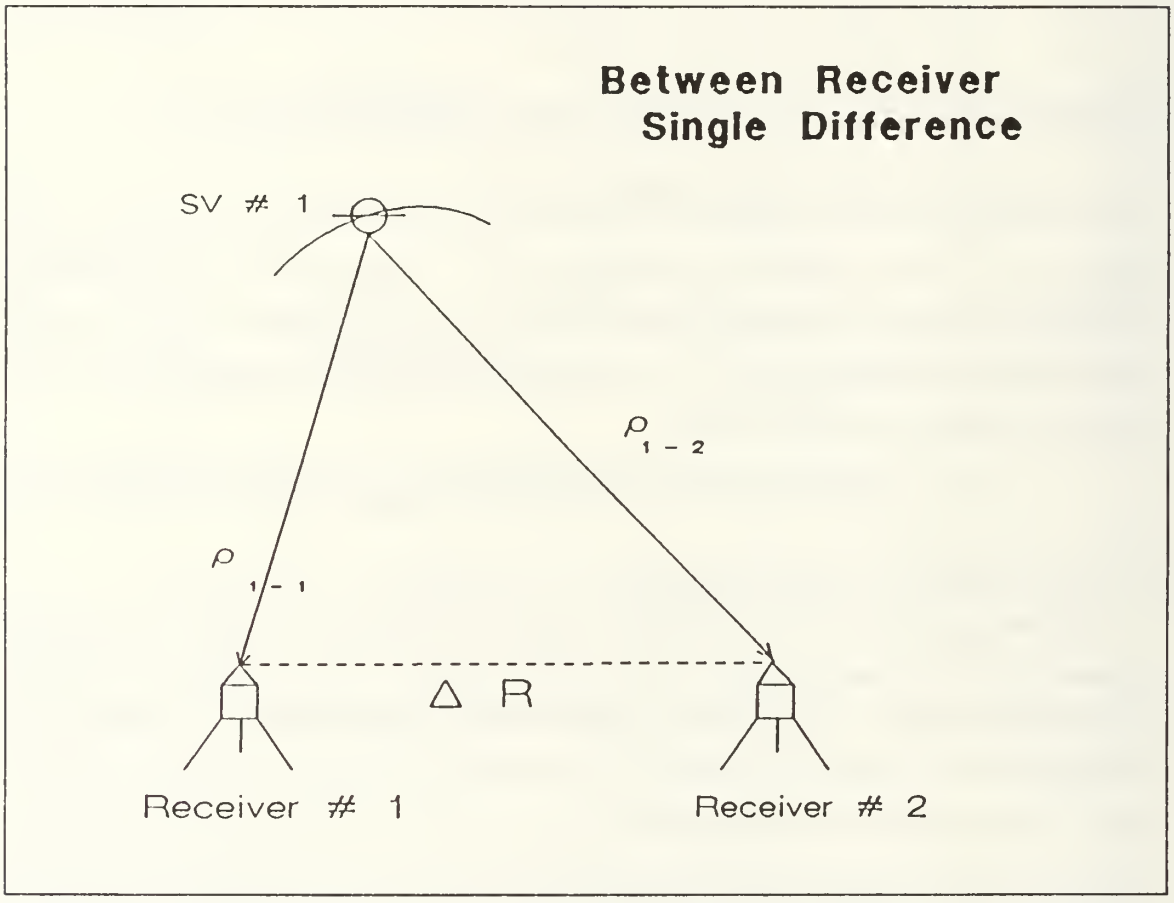

Figure 5. Single difference between receivers.: ΔR is the vector between the two receivers, sv is the "space vehicle" (satellite) and ρ is the measured range.

In kinematic GPS the first two single differences described are the ones most used. Considering that one of the receivers is in continuous motion, differencing between epochs is not particularly useful. From the phase biased range observation equation, which we have previously developed, it is easy to see the effects of differencing between receivers.

$$(\rho_\phi)^i_\alpha = \rho^i_\alpha + c\ \tau_i - c\tau_\alpha + \lambda N^i_\alpha - d_{ion} + d_{trop} + noise$$
$$(\rho_\phi)^i_\beta = \rho^i_\beta + c\ \tau_i - c\,\tau_\beta + \lambda N^i_\beta - d_{ion} + d_{trop} + noise$$

Taking the difference between these two equations eliminates the common parameter - the satellite clock offset (τ_s) - while considerably reducing the ionospheric and tropospheric effects.

$$(\Delta\rho_\phi)^i_{\alpha-\beta} = (\rho^i_\alpha - \rho^i_\beta) - c(\tau_\alpha - \tau_\beta) + \lambda(N^i_\alpha - N^i_\beta) - \Delta d_{ion} + \Delta d_{trop} + \Delta noise$$

This is the differencing technique used in " Differential GPS " where one receiver is fixed in a known position and the second receiver is either stationary at an unknown site or moving as in kinematic GPS. This technique is dependent on our knowledge of one receiver's position. We lose the ability to find the absolute position of the mobile receiver and can only find the relative position, with regard to the reference receiver. If we combine two kinds of single differences we obtain the double difference which is the basis of kinematic GPS.

2. Double Differences

Although any combination of single differences produces a double difference, the common pair used is the between - receiver and between - satellite pair. The order in which the differencing is computed does not affect the results since differencing is a linear procedure. In the example described we arrived at an equation for the between - receiver single difference. Writing two such equations, each for a separate receiver - satellite pair results in the following :

$$(\Delta\rho_\phi)^l_{\alpha-\beta} = (\rho^l_\alpha - \rho^l_\beta) - c(\tau_\alpha - \tau_\beta) + \lambda(N^l_\alpha - N^l_\beta) - \Delta d_{lon} + \Delta d_{trop} + \Delta noise$$
$$(\Delta\rho_\phi)^j_{\alpha-\beta} = (\rho^j_\alpha - \rho^j_\beta) - c(\tau_\alpha - \tau_\beta) + \lambda(N^j_\alpha - N^j_\beta) - \Delta d_{ion} + \Delta d_{trop} + \Delta noise$$

Differencing the two equations results in the double difference equation,

$$(\Delta^2\rho_\phi)^{i-j}_{\alpha-\beta} = (\rho^i_\alpha - \rho^i_\beta - \rho^j_\alpha + \rho^j_\beta) + \lambda(N^i_\alpha - N^i_\beta - N^j_\alpha + N^j_\beta) - \Delta^2_{ion} + \Delta^2_{trop} + \Delta^2_{noise}$$

It is evident that in this case both the receiver and the satellite clock errors have been eliminated. In this equation,

$$N^{i-j}_{\alpha-\beta} = (N^i_\alpha - N^i_\beta - N^j_\alpha + N^j_\beta)$$

is the integer ambiguity which has to be solved for,

$$(\rho_\alpha^i - \rho_\beta^i - \rho_\alpha^j + \rho_\beta^j)$$

contains the true distances between the various receiver satellite pairs where the two satellite positions are considered known, one receiver position is known and the second receiver position is the unknown. The term $(\Delta^2\rho_\phi)_{\alpha-\beta}^{i-j}$ is readily obtained from the respective phase measurements of the receivers.

The above double difference equation refers to measurements taken at a single epoch. The receiver at the fixed site is called the reference receiver while one satellite, with which all the others are differenced, is commonly called the reference satellite. Usually the reference satellite is determined in the post-processing procedure according to the highest elevation angle obtained and the longest visibility record of all the satellites during the session. This is the double difference equation used to determine the mobile receiver's position. Figure 6 shows the between - receiver between - satellite double difference.

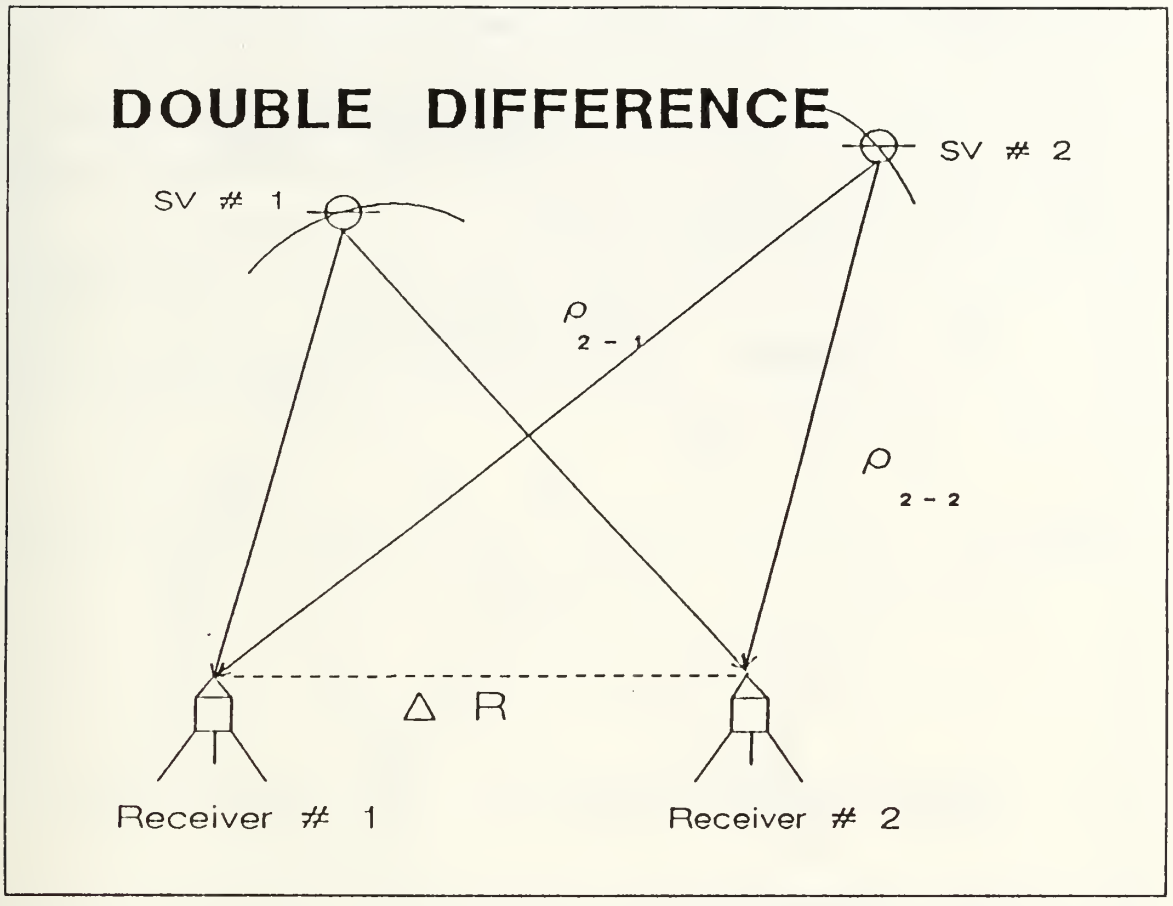

Figure 6. Between - receiver between - satellite double difference.: ΔR is the vector between the two receivers, sv is the "space vehicle" (satellite) and ρ is the measured range.

3. Triple Differences

The receiver - satellite - time triple difference is the change in the double difference described above from one epoch to the next. The resulting equation is:

$$(\Delta^3 \rho_\phi)^{i-j}_{\alpha-\beta,\,1-2} = \delta^3 \rho - \delta^3 d_{ion} + \delta^3 d_{trop}$$

where the latter two terms are now negligible, the initial integer ambiguities, common to both measurements, have been eliminated and

$$\delta^3 \rho = (\rho^i_{\alpha,1} - \rho^i_{\beta,1} - \rho^j_{\alpha,1} + \rho^j_{\beta,1}) - (\rho^i_{\alpha,2} - \rho^i_{\beta,2} - \rho^j_{\alpha,2} + \rho^j_{\beta,2})$$

Undifferenced observations are affected by biases in receiver clocks, satellite clocks and, in the case of carrier beat phase observations, initial cycle ambiguities. These have been eliminated in the triple differencing process. However, the number of measurements available has been reduced considerably - eight separate measurements, two receivers, two satellites and at two epochs, make up one triple difference observable. Figure 7 shows the triple difference procedure.

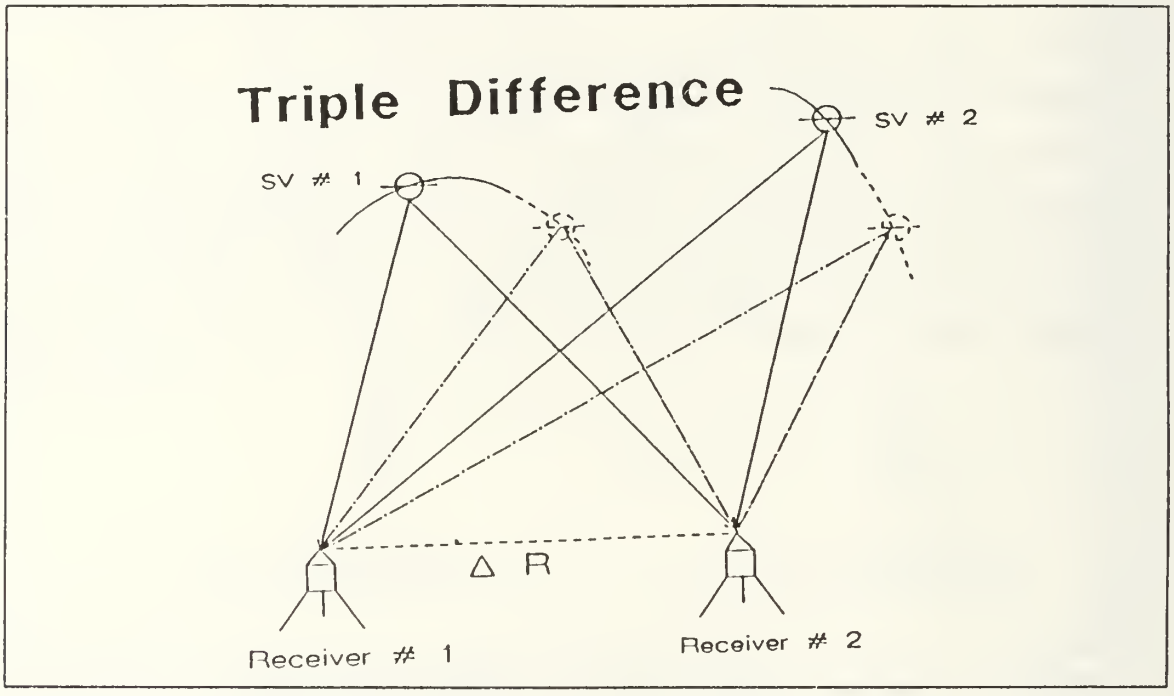

Figure 7. Triple difference procedure: ΔR is the vector between the two receivers and sv the "space vehicle" (satellite)

E. GPS SATELLITE GEOMETRY AND ACCURACY.

The accuracy with which positions are determined using GPS depends on two factors: the range measurement accuracy and the satellite geometry. The effect of the latter is expressed by the " dilution of precision " or DOP factor, which is the ratio between the positioning accuracy and the measurement accuracy. The DOP therefore is a measure of the geometrical " strength " of the satellite configuration, which varies with time as the satellites move along their orbits. The DOP factors are obtained from the combinations of the diagonal elements of the covariance matrix of the adjusted parameters. Figure 8 illustrates the geometrical relationship between poor and good DOPs.

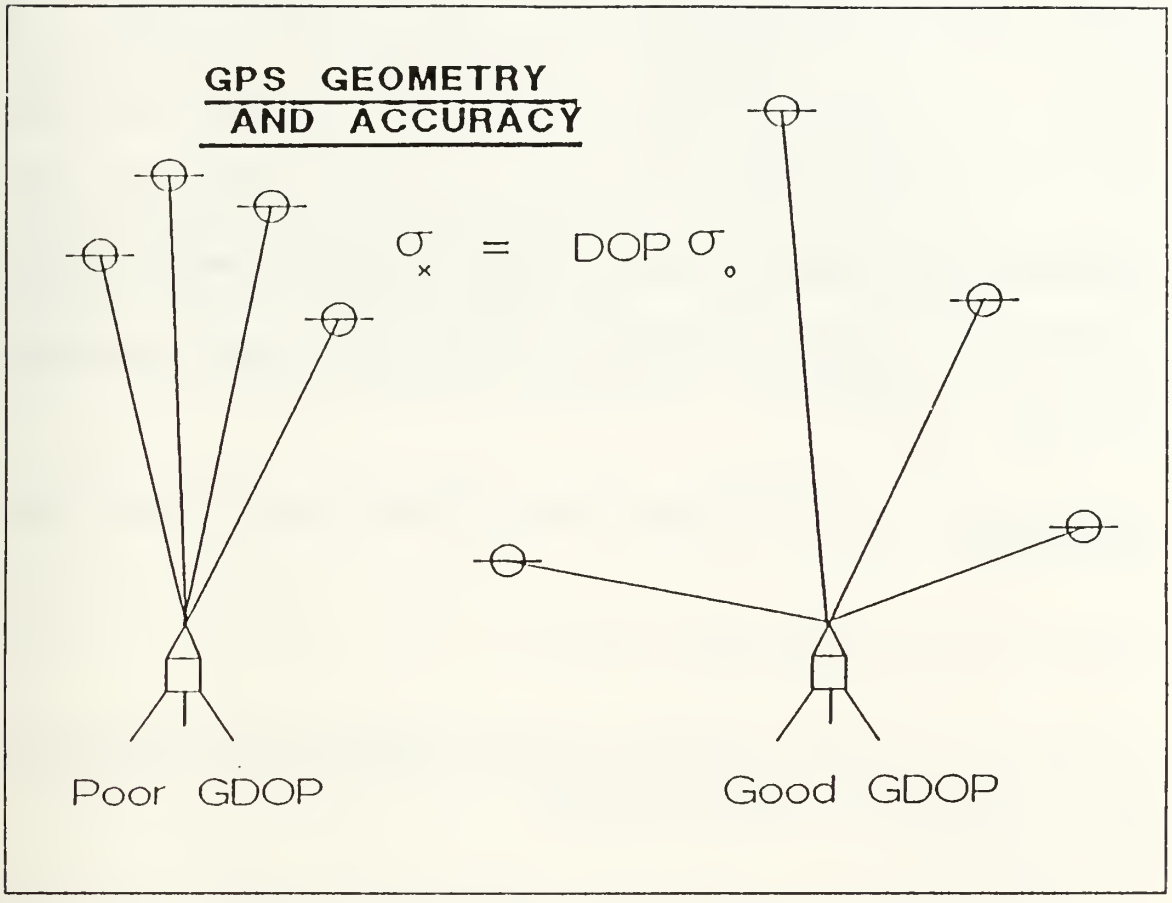

Figure 8. GPS Geometry and Accuracy.

The positioning accuracy is roughly given by the equation,

$$\sigma_x = DOP \cdot \sigma_0$$

where σ_0 is the noise in the measurement and σ_x is the position accuracy. The position accuracy can mean either the standard deviation of the position in one dimension or in a combination of dimensions. This leads to a variety of DOPs commonly referred to in GPS. Using a local north, east and up coordinate system, the covariance matrix of the adjusted parameters of the form :

$$\begin{bmatrix} \sigma_n^2 & \sigma_{ne} & \sigma_{nh} & \sigma_{nt} \\ \sigma_{en} & \sigma_e^2 & \sigma_{eh} & \sigma_{et} \\ \sigma_{hn} & \sigma_{he} & \sigma_h^2 & \sigma_{ht} \\ \sigma_{tn} & \sigma_{te} & \sigma_{th} & \sigma_t^2 \end{bmatrix}$$

VDOP refers to the vertical component or standard deviation in the height - σ_h.

HDOP refers to the horizontal component in two dimensions - such as north and east for example - $\sqrt{\sigma_n^2 + \sigma_e^2}$.

PDOP refers to the three dimensional position component without the time factor - $\sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_h^2}$.

TDOP refers to the time bias factor only - σ_t.

GDOP refers to the geometrical dilution of precision including the three dimensional and time bias components - $\sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_h^2 + \sigma_t^2}$.

F. MULTIPATH

Multipath is the phenomenon where a signal arrives at an antenna via two or more different paths. The difference in path lengths causes the signals to interfere at the antenna. A ship's superstructure is an ideal environment for multipath. In shipborne kinematic GPS surveying multipath can cause frequent loss of lock and cycle slips.

In the MBPPE great caution was taken when planning the GPS antenna installation on the ship's mast. The site chosen, at the top of the crow's nest, seemed to be the best option available. Results of the experiment show that the choice was a good one since relatively few cycle slips occurred on either day.

G. SELECTIVE AVAILABILITY - SA.

The Department of Defense (DoD) has a mandate for determining accuracy levels of GPS available to non - DoD users. The present policy specifies that Standard Positioning Service (SPS) will be available worldwide at the 100 m level when the system is fully operational. Precise Positioning Service (PPS) with a higher accuracy remains restricted to U.S. and allied military and to specialized U.S non - military uses which can be shown to be in the national interest.

There are basically two ways to degrade GPS accuracy. Selective Availability introduces clock errors by deliberately degrading the stability of the on - board atomic clocks and by degrading the navigation message transmitted by the satellites. Anti Spoofing (AS) implies that the P - codes transmitted by the satellites will be encrypted. Most receivers used in surveying are C / A - code receivers and will therefore not be affected by AS. Differencing the satellite signals between two receivers, as described in the differencing procedures, eliminates any satellite clock errors. Therefore, using the differential mode of surveying for relative positioning, with baselines of less than 100 km, will eliminate the adverse effects of Selective Availability.

III. KINEMATIC GPS

A. CONCEPTS

Kinematic positioning refers to the determination of a trajectory of a moving object with a GPS receiver and antenna mounted on it. Kinematic positioning, though related to navigation, is somewhat different. For navigation, real-time, less accurate positions are needed with speed and heading information being important. For most kinematic GPS applications, highly accurate positions are sought with post-mission results being usually acceptable.

Absolute kinematic positioning refers to the positioning of a moving receiver with respect to a specified coordinate system. This mode of positioning involves determination of the vectors between the origin of the coordinate system (usually the geocenter) and the instantaneous receiver positions. Since this vector is large, the accuracy achievable is limited. This type of kinematic surveying is not commonly used.

Relative kinematic positioning refers to the positioning of a mobile receiver relative to a stationary one, with both receivers observing the same satellites. The vectors determined are much shorter than for absolute positioning. They originate at the fixed, known site and terminate at the instantaneous mobile receiver's position. Many of the errors in measuring the ranges to the various satellites are common to both receivers and therefore cancel out. This produces the most accurate GPS mode of kinematic positioning and thus the one most often used. The MBPPE involved GPS kinematic surveying in the relative mode.

1. Kinematic Positioning Models.

Four types of models are used for relative kinematic positioning. These include pseudo-range only, phase only, combined and integrated models.

In the pseudo-range only model, pseudo-range measurements to at least four satellites are differenced between the two receivers. The resulting observation equation can be written as $\Delta\rho_R = f_1\{\Delta R, \Delta\tau\}$. The mobile receiver's position is determined relative to the fixed receiver and the difference in the receiver clock biases is determined.

In the phase only model, phase measurements to at least four satellites are used to determine coordinate differences with respect to the known position of the fixed re-

ceiver. The resulting observation equation can be written as $\Delta\delta\rho_\phi = f_2\{\Delta\delta R, \Delta\delta\tau\}$ Here the clock drift between the two receivers is also solved for. The combined method uses both of the above methods for determining the moving receiver's position, clock biases and clock drift. The integrated method adds external data, such as from an inertial navigation system, to the GPS measurements. This is used to overcome cycle slips and other problems in the GPS data and provides the most accurate output available.

B. AMBIGUITY RESOLUTION.

As seen in the carrier phase observation equations, the unknowns we wish to solve for include three position states and the initial cycle counts or integer ambiguities. Once the receiver has locked on to the satellite signal, the integer cycle count is continued. Only at initialization or after a cycle slip, when the cycle count is lost, is it necessary to solve the integer ambiguity. The double differenced phase observation equation at time t_0 is given by the equation :

$$\Delta\phi^{ij}_{\alpha\beta}(R_2 - R_1, t_0) = \frac{-f}{c}\left[\rho^i_\alpha(R_1, t_0) - \rho^j_\alpha(R_1, t_0) - \rho^i_\beta(R_2, t_0) + \rho^j_\beta(R_2, t_0)\right] + N^{ij}_{\alpha\beta}$$

In this equation ρ is a function of the receiver - satellite pair and the instantaneous satellite and receiver coordinates. In the initializing procedure, with a known baseline input, the ranges ρ are known, $\Delta\phi^{ij}_{\alpha\beta}$ is measured and therefore $N^{ij}_{\alpha\beta}$ remains the only unknown. Using at least four measurements, which result in at least three double differences, we can determine the integer cycle ambiguities. Actually, the various ambiguity combinations will be determined according to the receiver - satellite pairs. It is usual to regard one satellite as a reference to which all other satellites are differenced. The reference satellite is chosen in most processing programs according to the amount of time it is tracked and the maximum elevation angle obtained.

Three main factors contribute to a non-integer ambiguity being found :

Imperfect knowledge of the mobile station's position at initialization.
Clock errors resulting in incorrect phase corrections.
Satellite position errors.

Ionospheric differences, particularly during the day and with baselines over 50 km long, can have a strong effect on the cycle ambiguity resolution. Integer ambiguity values of between 0.25 and 0.75 from an integer are considered bad. However, if necessary, they can be used and a solution forced. Most processing programs warn of bad integer am-

biguity resolution, but allow the user to decide whether or not to use the values obtained. There are essentially two methods of treating a non - integer ambiguity; either rounding it to the nearest integer or treating it as a floating point real value. Most programs suggest rounding to an integer value for better results.

1. **Initialization Techniques**

Basically there are three methods used for solving the initial integer ambiguities. Once solved for, the kinematic survey can commence with only the three positional coordinates of the mobile receiver being unknowns. Should a cycle slip occur, reinitialization is necessary if only four satellites are being tracked. This is not always practically possible. The three techniques described have both merits and limitations and the choice of one instead of the others is usually determined by the kind of kinematic survey being undertaken.

a. Static Survey Initialization.

One method for determining the cycle ambiguity is to run a static, differential survey for a period of time prior to going kinematic. During this period, a fair solution for the receiver's position can be obtained from a triple difference solution (where the integer cycle ambiguity is differenced out). Once a position is obtained the double difference technique, with the cycle ambiguity unknown, is implemented. In this case the integer ambiguities are solved for using regular point positioning techniques. Once enough data has been gathered for a good ambiguity resolution to be determined, the survey is transformed to the kinematic mode.

This method is good for land applications where the mobile receiver can be set at a fixed site for any length of time. However, when mounted on a ship, even in the calmest conditions, the antenna is going to move. This degrades the cycle ambiguity resolution since the initial baseline is moving around. Attempts to use this kind of initialization procedure for the experiment were successful. However the coordinates of the antenna on the ship, obtained from a 20 minute " static " survey, produced a questionable baseline for initialization. The post-processing programs enable relaxation of the constraint that the solved integer ambiguity be within 0.25 of an integer. Thus it is possible to use the integers obtained for initialization. However, the relatively poor integers used will propagate errors throughout the processing procedure.

b. Antenna Swap.

This technique takes advantage of the fact that one of the sites in the survey is known and that the instantaneous range to any satellite can easily be determined. This leaves the integer ambiguity as the only unknown in the equation. The technique how-

ever requires that both antennas be interchangeable without any obstructions causing cycle slips in the process. This technique, illustrated in figure 9, is usually only practical in surveys conducted on land.

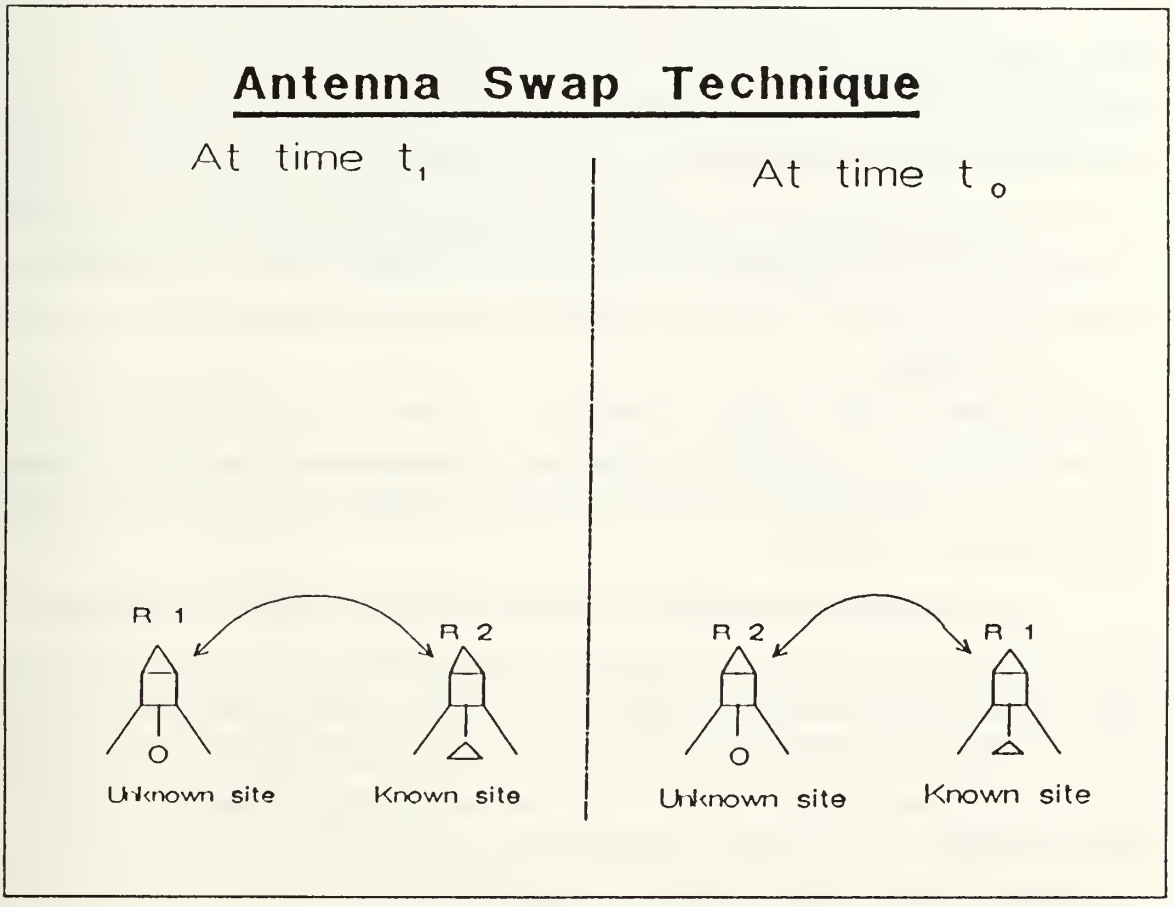

Figure 9. The Antenna Swap: Here R is the receiver, the circle symbolizes an unknown position and the triangle symbolizes a known position.

The antennas are set up, one on the known site and the other at an arbitrary unknown position, preferably not far away. After a short observation period (even a few seconds usually suffice), the antennas are swapped. The antenna which was on the known site has essentially solved the integer ambiguities while the second antenna now has a chance at using the fixed site to calculate its ambiguities. After a few more seconds of observation, the antennas are once again swapped back with the antenna at the fixed site now remaining as the reference antenna. The second antenna now is free to move and as long as lock is maintained on both receivers, reinitialization is not necessary. It

is quite apparent that this technique is totally impractical for use on a ship, where the second antenna is mounted on a mast.

c. External Baseline Input.

This is the method used in the MBPPE. An external measurement was made of the antenna position on the ship. Having determined the reference site's position the baseline was computed between the fixed shore site and the position of the GPS antenna on the ship's mast at the instant the latter was measured. This instantaneous baseline was used to solve the integer ambiguities and the results showed that for most satellite - receiver pairs, the ambiguities were very close to integer numbers. This technique is limited to relatively short baselines with positions known to within centimeter accuracy.

2. Cycle Slips.

When a satellite signal is obstructed in any way, it can no longer be tracked. When signal lock is resumed, the fractional part of the measured phase would still be the same as if tracking had been maintained. The integer number of cycles, however, exhibits a discontinuity - " cycle slip ".

There are various approaches to dealing with this problem. For differential surveying where both the receivers are stationary, the problem is less critical. However, for kinematic differential surveying, cycle slips can be instrumental in terminating the survey. Only when at least four satellites are being tracked, and a reasonable instantaneous position is assured, can cycle slips on any other satellite signals be repaired. This is accomplished in a similar fashion to the fixed baseline initialization with the computed position from the (at least) four good satellites used.

In certain post - processing programs, when less than four satellites maintain lock it is possible to try to model the data for each satellite - receiver pair where cycle slips occur. If a piecewise continuous polynomial fit to the cycle count is obtained, it may be possible to rectify the cycle slip and bridge the data gap. However, this is a tedious task involving manual examination of the data to find the breaks and perhaps manual editing at the few cycle level. Recalling that the cycle count is a function of the relative satellite - receiver motion, and the changing range between them, it is almost impossible to extrapolate precisely over a cycle slip. The satellite motion is relatively smooth, but the movement of a ship at sea certainly isn't. When the second receiver is stationary, and only the satellite is moving in the earth - centered earth - fixed coordinate system, it is easier to use extrapolation to attempt modeling the cycle count. In this case however, another method is available. By determining a triple difference solution, as in

the " static " survey initialization technique, it is possible to resume the survey after a cycle slip.

C. SOLVING FOR POSITION.

As mentioned before, solving for the mobile receiver's position coordinates is an iterative adjustment process. This process is conducted sequentially, at each measurement record, to obtain a trajectory of the receiver. In order to obtain the most accurate results, the double differenced phase observable is used.

The measured double differenced phase residual is given by the expression :

$$\Delta^2(\phi^{ij}_{\alpha\beta}) = \Delta\phi^i_\alpha - \Delta\phi^j_\alpha - \Delta\phi^i_\beta + \Delta\phi^j_\beta$$

where ϕ is the one way phase residual, α, β are the receivers and i , j are the satellites. This is in fact the L^b in the adjustment computation developed in the previous chapter. The theoretical, calculated double differenced phase value is given by :

$$\Delta^2(\phi^{ij}_{\alpha\beta})' = -\frac{f}{c}(\rho^i_\alpha - \rho^i_\beta - \rho^j_\alpha + \rho^j_\beta) + \Delta N^{ij}_{\alpha\beta} - \Delta^2_{ion} + \Delta^2_{trop} + \Delta^2_{noise}$$

In the above equation, with α being the known reference station and the satellites i and j having known coordinates, the unknown x, y and z coordinates of the mobile receiver enter into the ρ^i_β and ρ^j_β terms.

In the adjustment computations description (Appendix B), the theoretical values of the observed parameter were derived from a function of the unknown parameters - $L^a = F(X^o)$. The unknown parameters may be estimated and a Taylor series expansion about the estimated values made. When truncating this series to retain only linear terms, we get :

$$F(x^a) = F(X^0) + \frac{\partial F}{\partial X^a}\Big|_{x^a = x^0}(X^a - X^0)$$

Substituting in for X the mobile receiver's coordinates, x, y and z, and using estimated initial positions, x_0, y_0 and z_0 results in the equation :

$$\Delta^2\phi^{ij}_{\alpha\beta} = -\frac{f}{c}(\rho^{ij}_{\alpha\beta})_0 - \frac{f}{c}(\Delta\rho^{ij}_{\alpha\beta}) + ...$$

where $\rho^{ij}_{\alpha\beta_0}$ is the double differenced range between the satellites, the reference receiver and the mobile receiver with its estimated a priori position. $\Delta\rho^{ij}_{\alpha\beta}$ is the change in the

double differenced range due to the mobile receiver being slightly off the estimated position. Figure 10 shows the double differenced range between the two receivers and two satellites.

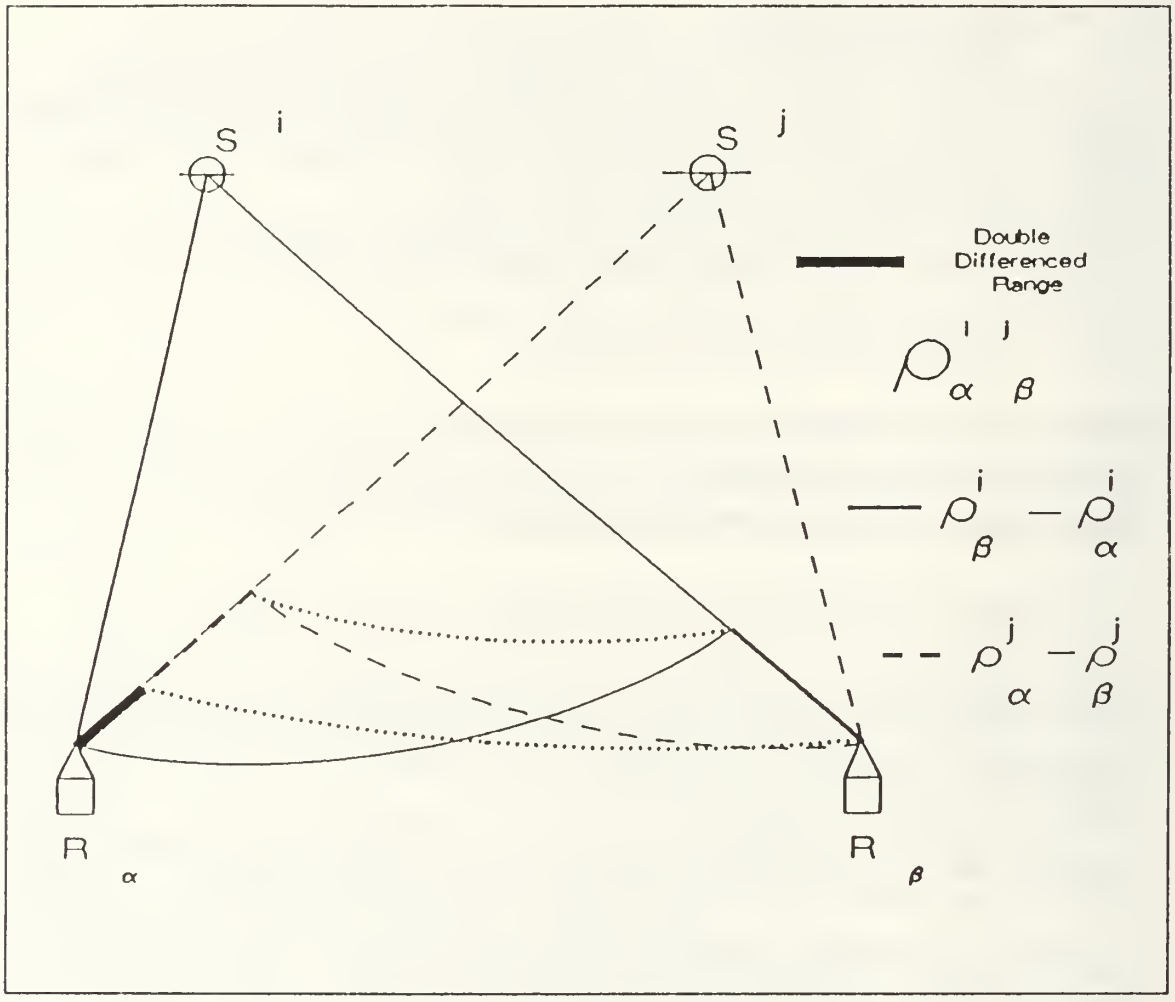

Figure 10. The double differenced range: This diagram shows the differencing of the satellite - receiver ranges and the relationship to the double differenced phase measurement.

In this figure the straight lines indicate the distance measurements, the curved lines imply that the distances are equal on both range lines. ρ is the calculated range between the satellite and receiver.

The above equation can now be written as :

$$(\Delta^2\phi^{ij}_{\alpha\beta}) = -\frac{f}{c}\left\{\left[\frac{x^i - x_\beta}{\rho^i_\beta} - \frac{x^j - x_\beta}{\rho^j_\beta}\right]\Delta x_\beta + \left[\frac{y^i - y_\beta}{\rho^i_\beta} - \frac{y^j - y_\beta}{\rho^j_\beta}\right]\Delta y_\beta + \ldots\right.$$

$$\left. \ldots + \left[\frac{z^i - z_\beta}{\rho^i_\beta} - \frac{z^j - z_\beta}{\rho^j_\beta}\right]\Delta z_\beta\right\} + \Delta^2 N^{ij}_{\alpha\beta} + Noise$$

The next step is to subtract this calculated value of the observable from the measured value to obtain a residual. This residual will be added to the estimated parameters, x_0, y_0 and z_0 , to get new estimates which will be used in the next iteration. In the initialization we assume the x_0, y_0 and z_0 values are precisely enough known and that Δx, Δy and Δz all equal zero. One double difference phase measurement therefore solves for the cycle ambiguity $\Delta^2 N^{ij}_{\alpha\beta}$ for the relevant receiver and satellite set. Once $\Delta^2 N^{ij}_{\alpha\beta}$ has been determined, from the next measurement on it is considered known and the remaining three parameters to be solved for are the Δx, Δy and Δz.

Three such equations, for three receiver - satellite double difference sets, with the common unknowns - mobile receiver offsets in the three coordinate directions - are sufficient to solve the equations. If we have R receivers and S satellites tracked, the number of independent double difference measurements available is (R - 1) (S - 1). With this in mind it is apparent that with R equal two, S must equal at least four to get three independant double differences. If more than four satellites are being tracked, we have redundancy in the system which strengthens the solution. The more the degrees of freedom, measurements minus unknowns, the stronger the solution. It is quite apparent that this method, while highly precise, has its limitations and weaknesses, especially when cycle slips occur and the $\Delta^2 N$ cycle ambiguity is lost.

IV. THE MONTEREY BAY PRECISE POSITIONING EXPERIMENT. (MBPPE)

A. GENERAL.

An experiment in the use of various GPS receivers on a ship was conducted on the Research Vessel POINT SUR during the week of December 3, 1990. The general purpose of the experiment was to gather data sets from the various receivers, along with a sub - meter reference trajectory, for evaluation of the receivers and the postprocessing techniques. The experiment was planned to allow for data collection for about six hours per session during which a maximum number of satellites could be tracked. For the Monterey Bay area, where the experiment was conducted, this meant collecting data from about 11:00 pm till 5:00 am. local time (6:00 am.- 12:00 noon UTC). Though not intentionally planned, this time period coincides with the time when ionospheric effects on the measurements are minimal. Between four and eight satellites were tracked simultaneously during each session. As an example, satellite tracking times for day 341, as observed by the Ashtech receiver on the ship, are shown in figure 11.

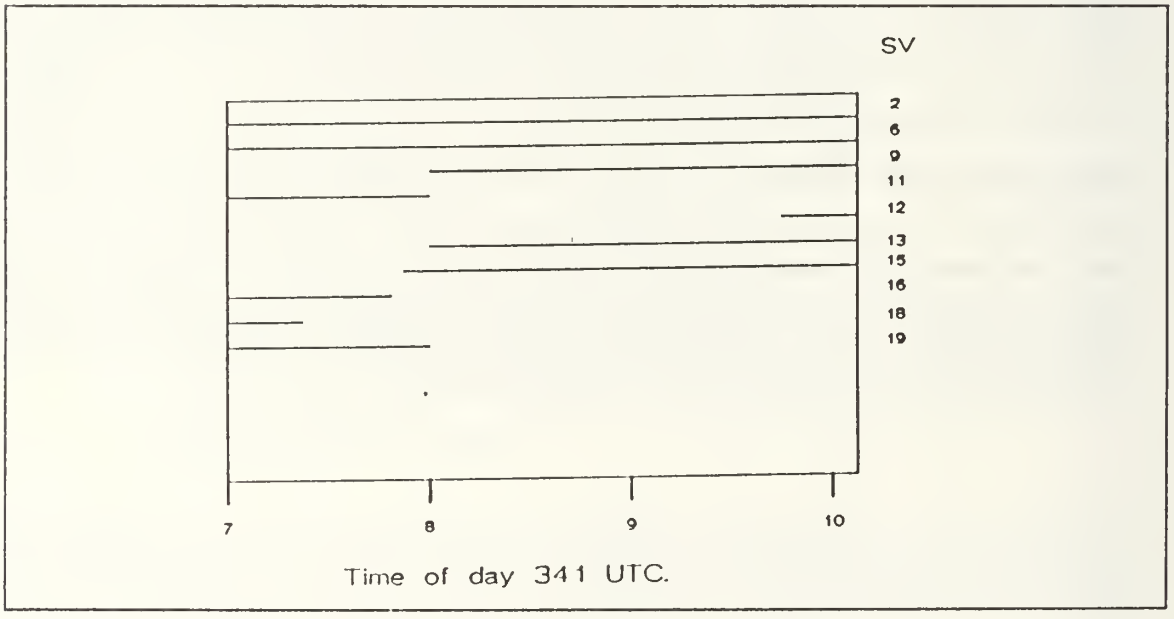

Figure 11. Satellite tracking times for day 341 - Ashtech.

Figure 12 shows the general location where the experiment was conducted.

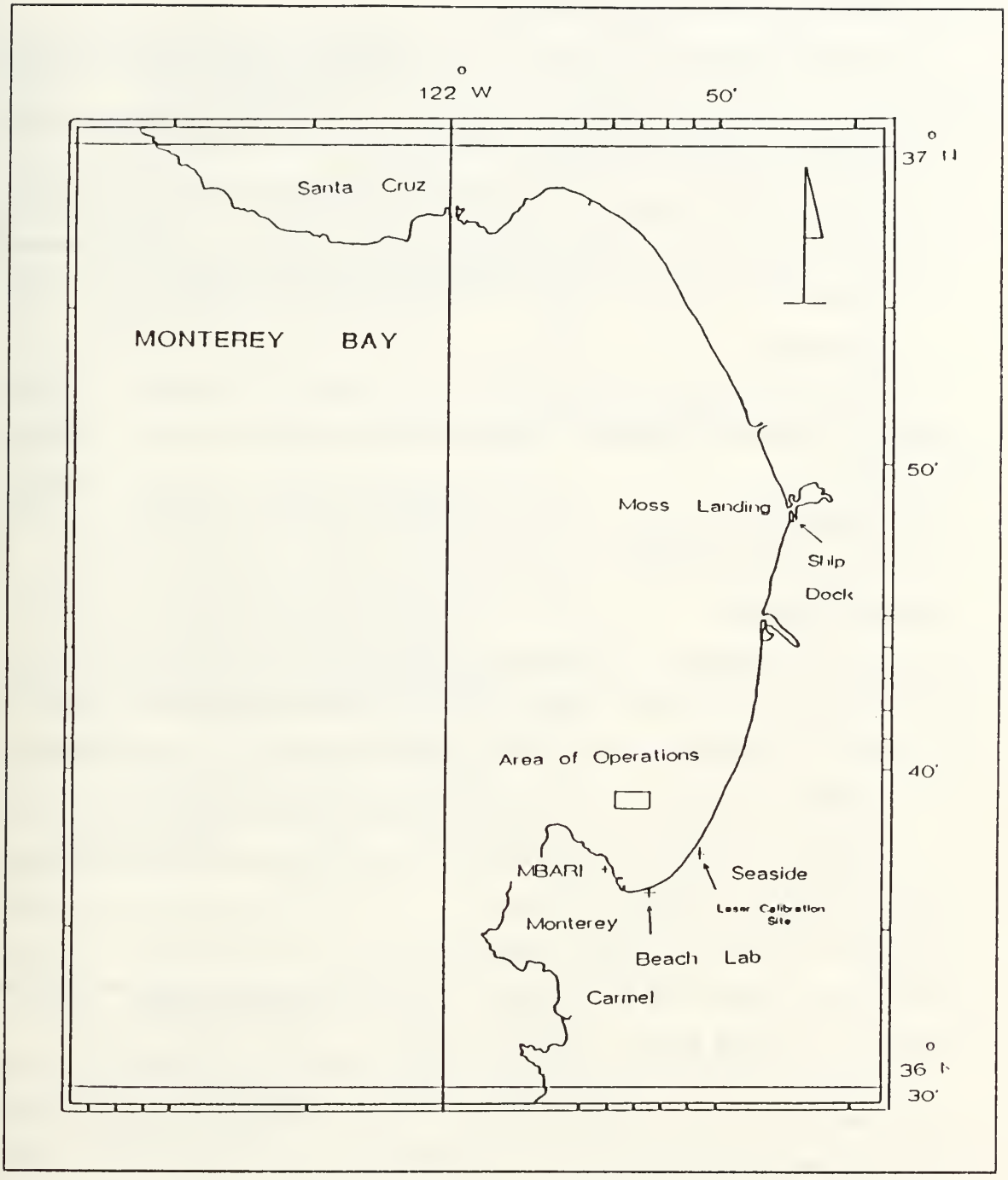

Figure 12. Map of the experiment area

B. OBJECTIVES OF THE EXPERIMENT

The stated objectives of the experiment were manifold. The ones listed here are only the objectives pertaining to the kinematic aspect of the experiment.

1. Evaluation of the post processing accuracies of both position and velocity utilizing both range and phase corrections.
2. Evaluation of direct kinematic solutions for the position and velocity of the ship.
3. Evaluation of specific shipboard limitations, such as multipath effects for example.
4. Evaluation of receiver clock quality effects on solutions of various types.

C. EQUIPMENT

The equipment used in this experiment can be categorized according to the function of each item. The three functional categories which distinguish one piece of equipment from another are:

GPS equipment

Reference Systems equipment

Geodetic equipment

The Ashtech and Trimble receivers used in the GPS part of the experiment were used to survey in some of the reference marks on the shore. Since this paper deals mainly with the Ashtech and Trimble receivers, a more detailed description of these instruments is given in tables 3, 4 and 5.

The Ashtech receivers used were the LD XII models with version 5 E software including many of the optional features offered. This included 12 all in view channels for satellite tracking, 6 MB internal memory and a one second recording interval option. The antenna used was the Ashtech geodetic microstrip antenna housed on a precision platform with a built-in vial and compass. This type of precision antenna is considered necessary in achieving geodetic standard accuracies.

The Trimble receivers used were eight channel, dual frequency receivers with 1 MB internal memory. An Office Support Module (O.S.M.) was used to connect the receiver to external equipment such as an AC power source and a PC used for data - logging. The high precision, dual frequency, geodetic microstrip antenna with the large groundplane used, provides highest accuracy GPS measurements.

D. METHOD

The experiment involved three principal phases: planning and preparation, data acquisition and data processing and analysis. The data acquisition stage included two

Table 3. EQUIPMENT USED

A	GPS Equipment
1	Two Ashtech LD XII dual frequency receivers
2	Two Trimble 4000 SST dual frequency receivers
3	Two Magnavox MX 4200 receivers
4	Two TI 4100 receivers with Rb Oscillators

B	Reference System Equipment
1	One Krupp Atlas Polartrack range and azimuth laser positioning system
2	One Krupp Atlas Polarfix range and azimuth laser positioning system
3	Four reflectors for the laser systems
4	A two - axis gyro system installed on the ship
5	A heading gyro installed on the ship
6	Motorola Mini - Ranger radio positioning system

C	Geodetic Equipment
1	Wild T 2000 theodolite
2	Wild DI 3000 EDM
3	Lufkin steel tape

parts, two sessions of data collection with the ship tied up to the pier and two sessions with the ship underway.

1. Planning and preparation

This part of the experiment included the logistical aspect of receiving all the equipment, checking it and learning how to use it. The sites where the equipment was

Table 4. FEATURES OF ASHTECH RECEIVER

Channels	12
Internal RAM capacity	6 MB
Recording Interval	1 second or more
Frequencies	Dual Frequency L1/L2
Software Package	GPPS version 3.0

Table 5. FEATURES OF TRIMBLE RECEIVER

Channels	8
Internal RAM capacity	1 MB - external data-logger used
Recording Interval	1 second or more
Frequencies	Dual Frequency L1/L2
Software Package	TRIMVEC - PLUS revision C

to be deployed, both on the ship and on the shore, were prepared. This involved both setting survey marks in the ground at the shore sites and installing poles on the crows nest of the ship on which the various antennas could be easily mounted. Geodetic measurements were then taken at each site to obtain the positions of the reference marks for the fixed receivers on shore and to get the relative positions of the various antennas in a ship - fixed coordinate system. At the end of the experiment another set of measurements were taken on the ship to account for any movement of the antannas relative to one another, during the week of the experiment.

The POINT SUR is a 300 ton ship, 45 m in length and with a maximum speed of 12 knots. The mast is about 17 m tall with a crow's nest on top. The crow's nest is surrounded by a metal railing about 1m high. Poles, about 2 m long, were attached so that they protruded about a meter above the top of the railing. The poles had been prepared to allow for the antennas to be easily, but steadfastly attached. With the antennas being so high up above the main deck, where the receivers were arranged in the " dry - lab " , 30 m antenna cables were needed. The two - axis gyro and the heading gyro system were also deployed in the " dry - lab " where they could be viewed, through

the open door of the lab, from the pier. This was necessary for the reference measurements taken prior to and after sailing. Figure 13 shows a side view of the ship POINT SUR with a top view of the crows nest and the antenna farm setup.

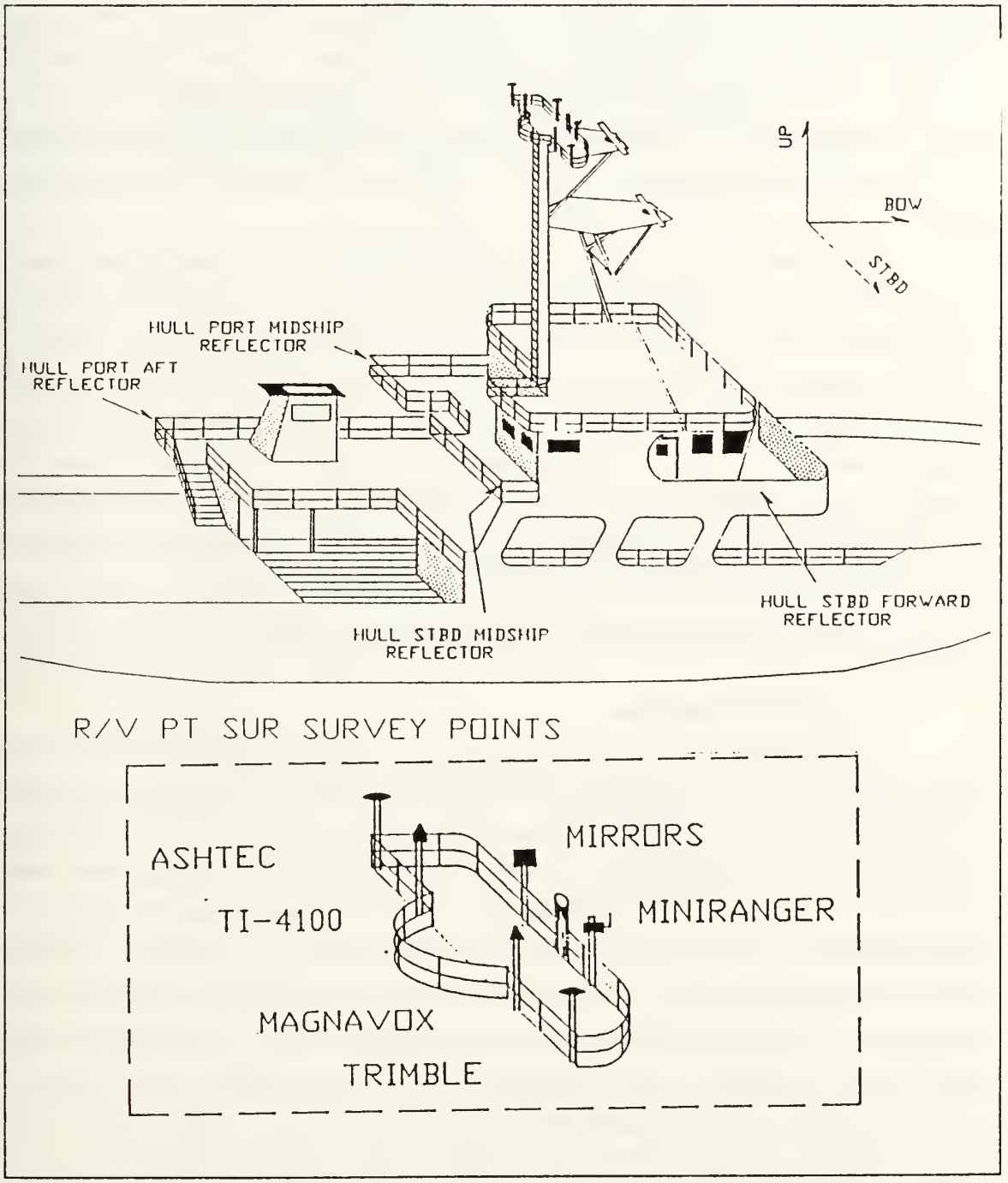

Figure 13. The POINT SUR

Survey marks were also set on the pier where the ship tied up so that measurements of the antenna array and two - axis gyro system could be taken prior to the ship's departure and after its return to the dock. These measurements produced a set of data points which fixed the various antenna positions in either geodetic, ship - fixed or earth centered - earth fixed coordinates. These positions, at known instants of time, were used to initialize the kinematic GPS surveys. The sites where the two Krupp Atlas laser systems were deployed and a third azimuth reference mark for their calibration, were surveyed and tied into the network. Figures 14, 15 and 16 show maps of the three survey sites.

These surveys were tied into a geodetic network for the Monterey Bay area and the coordinates of the ground stations used in the experiment were calculated [Ref. 7]. The Beach - Lab station array contained the four reference receiver sites; Ashtech on DOP1, Trimble on DOP2, Magnavox on DOP4 and TI on DOP5. The Polartrack laser system was on DOP3. Included in the array are DOPPLER, the original first order Doppler site, and CSID, the azimuth reference station for the laser systems. The MBARI array includes a presurveyed mark, MBARI and the new mark where the Polarfix system was set up, MBARI 1. The LOBOS array includes three marks on the pier where the ship docks. LOBOS is the original presurveyed mark and LOBOS 2 and LOBOS 3 were set for the experiment. This concluded the preparatory phase of the experiment.

2. In Port Reference Data

On the two days that the ship remained tied up to the pier, no reference data was collected. The laser systems were out of range and since the main motion of the ship was in the vertical, with the slow change in the tide, a reference " trajectory " was not necessary. The horizontal motion of the ship was on the order of a few decimeters which is within the noise level of all the reference systems. The main purpose of this phase of the experiment was to provide control for the "Kinematic" phase. The receivers were set up at the two sites, Beach - Lab and Ship, and data was collected for about six hours per session. This phase of the experiment was utilized to learn the receiver setup and downloading procedures and to discover any unforeseen problems in the planned procedure. Satellite rise and set times were noted to adjust the survey times planned for the two " seagoing " days. The data in the receivers which logged data internally, such as Ashtech, was downloaded to a PC after each session to enable the receiver's memory to be cleared. For receivers which were linked to an external data logger (PC), such as Trimble, the above procedure was not necessary. However a check was made at the end

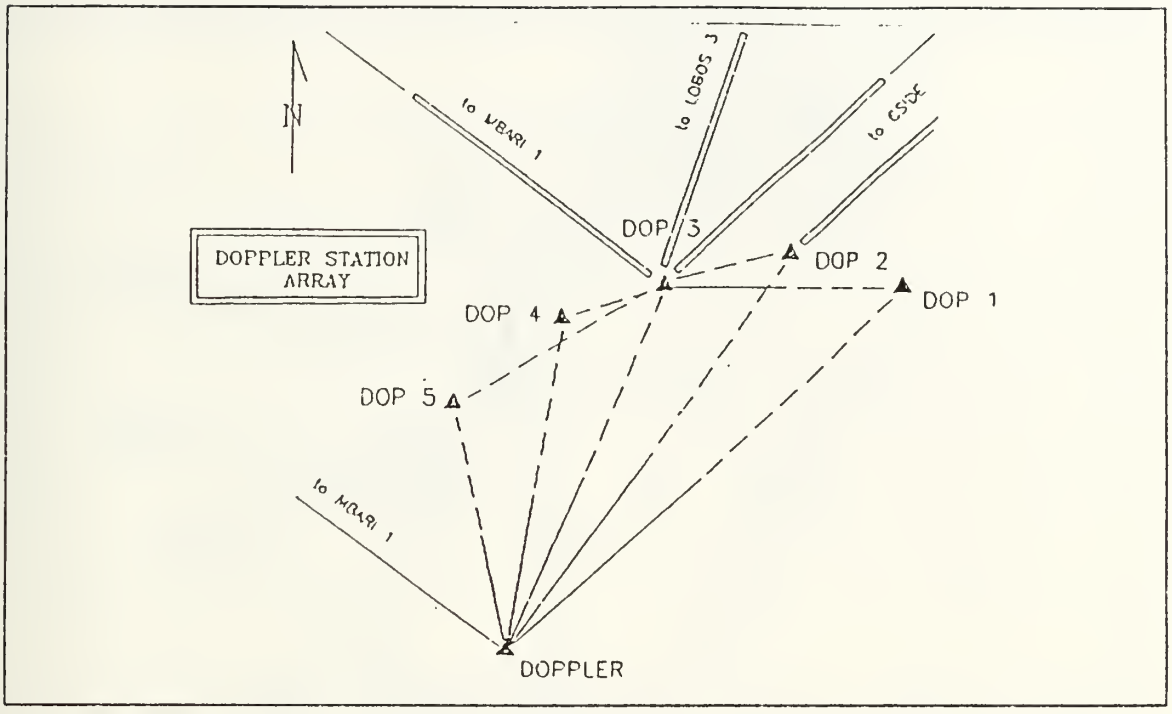

Figure 14. Beach - Lab station array: The map shows where the GPS antennas were deployed. The DOP sites were established about five meters apart.

of each session to ensure that the PC had enough memory left for data logging during the next session.

3. At Sea Data

The next phase of the experiment was to collect data with the ship underway and the reference systems in operation. Once again the receivers were set up on the shore and on the ship. The ship remained tied up to the pier for 20 to 30 minutes with all systems recording data. Measurements were taken from the pier to the antenna array and the two - axis gyro system, using the Theodolite and the EDM equipment. Once these measurements had been completed and all systems were ready, the ship left the dock and proceeded to the area, in the southern part of Monterey Bay, where the laser systems could track the ship's movement. During the transit when ship was out of range of the laser systems, for an hour after leaving the dockside until arriving at the designated survey area, all systems except the laser systems recorded data continuously. Prior to the ship coming within range of the laser systems, the two instruments were initialized on the two azimuth reference marks - the independent site at CSID about 3.5 km away

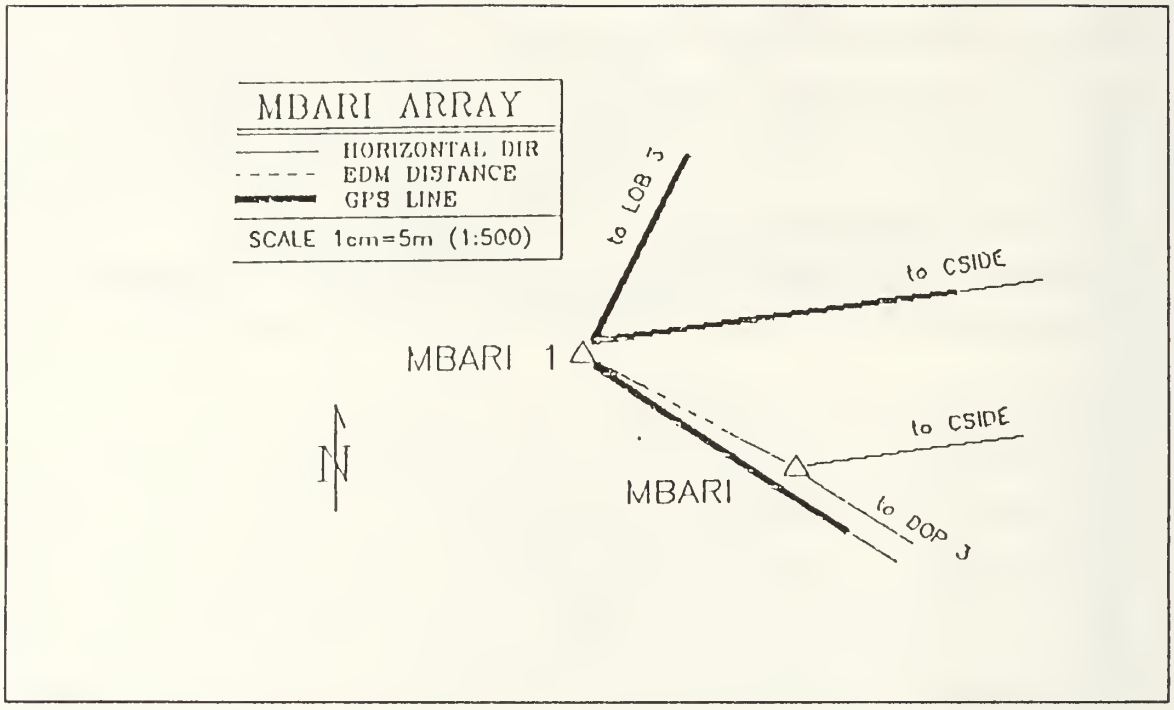

Figure 15. Map of MBARI array

and on each other. Then they were pointed towards the ship and upon locking on to the reflector situated on the crow's nest, they began tracking the ship's movement. The reference data recorded was logged on PC's linked to each laser system.

The ship then proceeded to sail along a designated route in the bay, changing speed and direction from time to time. Figure 17 describes the sailing pattern of the ship, as determined by the Ashtech processing for day 341. The ship sailed along an ellipse shaped track, going around three times while varying its speed between one, five and nine knots. This phase when all systems were simultaneously recording data lasted about three hours. The laser systems reinitialized by breaking off their automatic tracking and pointing to the azimuth reference mark twice more. This procedure, at the middle of the tracking session and after the ship was out of range on its return to port, ensured good calibration of the reference systems. They also caused a small (few minutes) gap in the reference data, but this did not affect the results in any way. The ship then returned to the dock where, after tying up, more measurements were made of the antenna array and the two - axis gyro system from the pier. During this period, for about

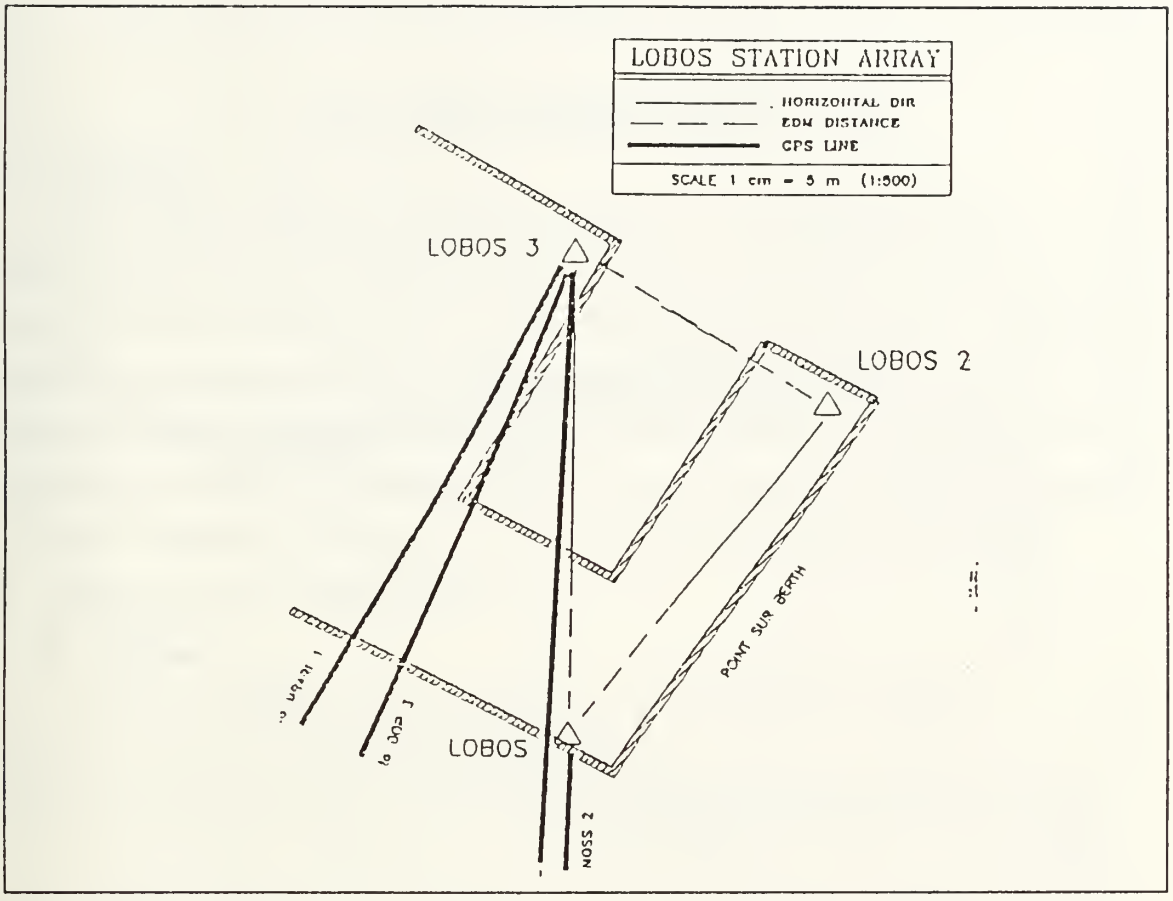

Figure 16. Map of LOBOS array

20 to 30 minutes, all GPS receivers continued to record data. Once again the data was then downloaded after each session to prepare the receivers for the next session.

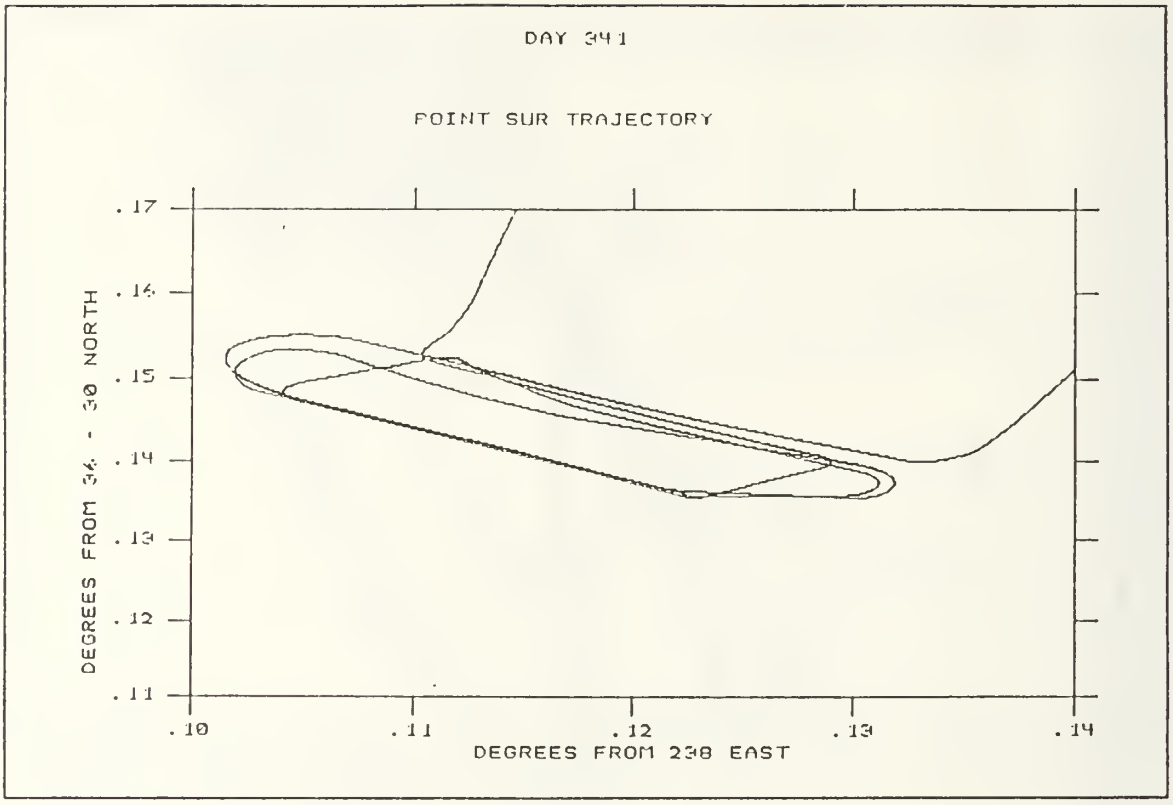

Figure 17. POINT SUR sailing pattern.

E. GPS DATA.

The information presented here is only a summary of the Ashtech and Trimble data collected during the experiment. Four data files were downloaded from the Ashtech receivers and four from the Trimble receivers. These files were immediately copied for backup. The large data sets, resulting from data acquisition at a one second rate for over six hours at a time, called for about 120 MB of disk space for storage purposes only. To facilitate this requirement three 44 MB Bernoulli disks were used for data storage. Appendix C contains tables with the data file names and sizes.

Once the data had been organized on disks, the processing phase could commence. A detailed description of the processing procedures and data analysis is presented in chapter V.

V. DATA PROCESSING PROCEDURES

A. GENERAL

This chapter describes the processing techniques used in obtaining the results presented. Both Ashtech and Trimble provide software packages for post-processing data collected by their GPS receivers while the National Geodetic Survey (NGS) has a general software package, OMNI, which can be used with a variety of receivers. There are both similarities and differences in the programs used by each vendor and these are summarized in table 4.1. One of the objectives of this study is to determine the effects of the post- processing programs on the accuracy levels obtained by each receiver.

B. ASHTECH DATA PROCESSING.

1. Method

Data processing involved a number of steps. The first step was to determine the periods of overlap for the two data sets collected each day (two sites with Ashtech receivers). A program was run on each of the four Ashtech data files providing a summary of times and number of observations made of each satellite by each receiver. A summary of periods during which one to eight simultaneous satellites were observed presented a basis for determining the times during which enough data was available for processing. For each session the minimum time span during which at least four satellites were observed simultaneously, at both sites (ship and shore), was determined. The reference trajectory was then examined to identify the critical period when the kinematic GPS trajectory could be evaluated to determine its accuracy. The times when geodetic measurements were taken of the antennas on the ship, while it was tied up to the dock prior to departure and after its return, were noted for the initialization of the kinematic phase of the survey.

The second step involved preparing the files necessary for manual processing of the data. Downloading Ashtech data from the receivers furnishes three files :

1. " B " files which contain the observed measurements.

2. " E " files which contain the Broadcast Ephemerises.

3. " S " files which contain the site and meteorological data input into the receiver during each session.

Figure 18 shows a flow chart of the Ashtech processing procedure. The program FILETOOL in the Ashtech GPPS package enables the user to browse through the data file (" B " file), to thin the file down with a larger epoch interval or to extract some of the data by specifying the first and last epochs desired. This program ran to extract data from the time the last geodetic measurement was taken of the Ashtech antenna prior to the ship's departure, until the last epoch when a minimum of four satellites were observed simultaneously. Next the program COMNAV was run to produce a COMMON.NAV file which is a file of the common Broadcast Ephemerises for the satellites observed for both sites. The program MAKEUFIL takes the " B " files and the COMMON.NAV file and produces " U " files which contain the undifferenced phase data required by the processing software. This was run for each receiver producing two " U " files per session (one per site). The program GENLOG takes the raw data " B " files and the COMMON.NAV file and outputs three ASCII files :

LOGTIMES - a file with information about the various stages of the survey, initialization, kinematic, occupation of known sites etc.

FILENAME.OBS - a file with the names of the " U " files which correspond to the raw data " B " files.

MARKPOS.ASC - a file with the position of the Fixed receiver and the site names and positions occupied by the Rover receiver during the kinematic survey and particularly at initialization.

The LOGTIMES file produced was then edited to indicate one data point of initialization at the reference measurement time and the rest of the session in continuous kinematic mode. The MARKPOS.ASC file was edited and the calculated positions of the fixed site on the shore and the Ashtech antenna on the ship, measured at initialization, were inserted.

The third step was to run the KINSRVY program which uses all the above files and calculates the ship receiver's position at each instant. The program outputs a summary file, KINSURVY.OUT and a trajectory file, ROVER.TRJ.

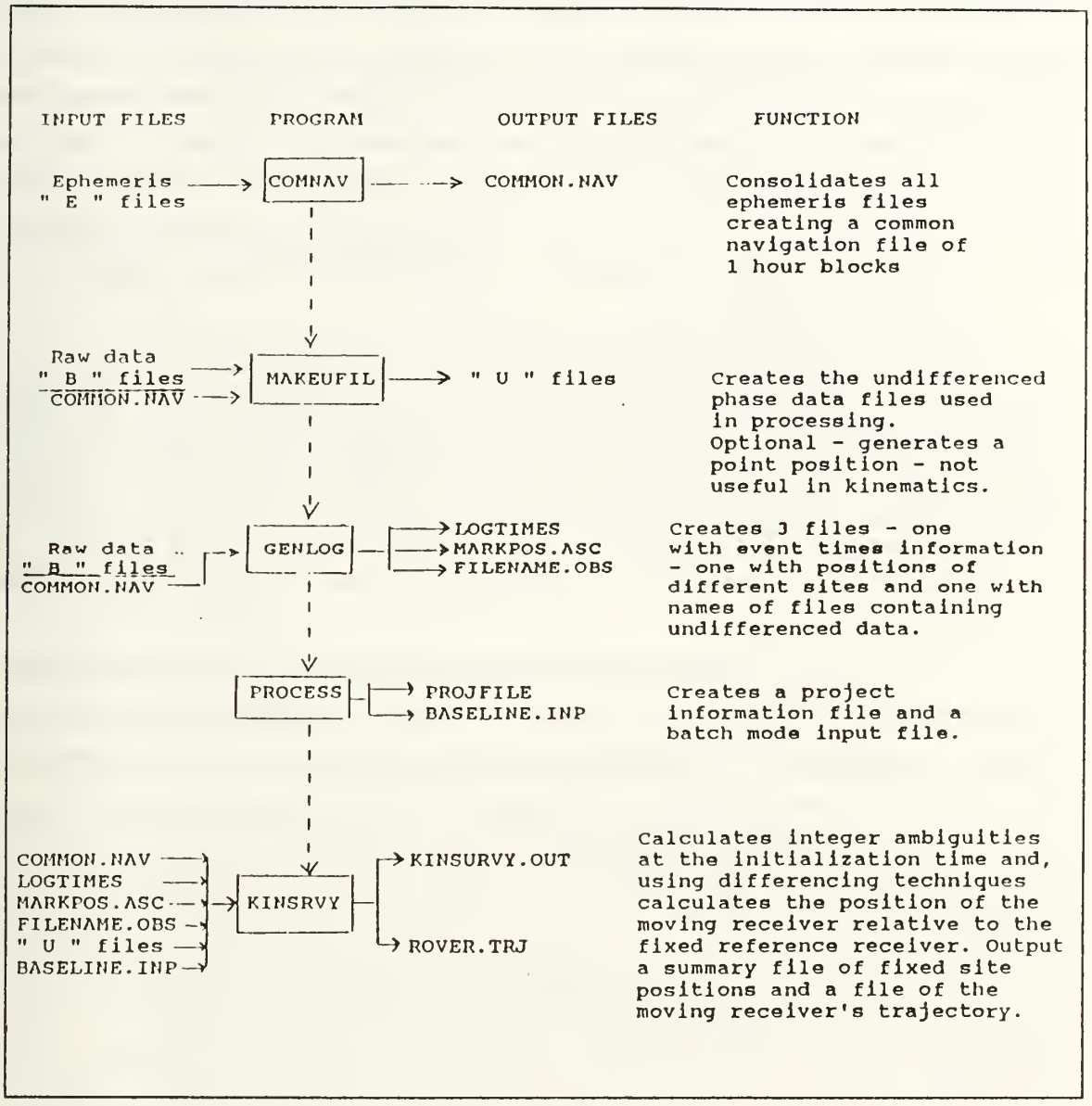

Figure 18. Ashtech data processing procedure.

2. Day 340 processing.

The KINSRVY program used on the day 340 data ran for about one and a half hours of data before it prematurely terminated. Examination of the raw data set revealed a seven second gap in the data, when no data was logged. At that time six satellites were being tracked. Four of the six lost lock and the phase biased range (cycle

count) was reinitialized. Only two satellites maintained a cycle count beyond the data gap. Continuous lock on a minimum of four satellites is a requirement for a kinematic solution and for cycle slips on other satellites to be repaired. Since this requirement was not met the program terminated. Attempts were made to bypass this gap in the data and reinitialize the kinematic survey while the ship was underway. The constraints for this type of procedure, where a high level of accuracy is required, are extremely stringent. None of the attempts made were successful in restarting the processing routine.

3. Day 341 processing.

Examination of the data indicated that there were no gaps in the data set as for the previous day. Once again the three step procedure, as described above, was applied to the data. The program ran for some time before terminating with a message indicating that less than four satellites were usable due to loss of lock or cycle slips. Examination of the raw data showed that while five satellites were being tracked, one was below the set elevation mask and one lost lock momentarily. This left only three usable satellites for processing at that instant.

There are two principal methods of processing the data, manually and automatically. Using the automatic processing technique would have enabled a simple change in the defaults of the PROJFILE file, including the elevation mask for processing of 15 degrees. However, automatic processing of the data necessitates the step where the " U " files are generated from the raw data " B " files. This results in four large files being in the directory at once. The raw data " B " files are not used in the processing and are therefore obsolete once the " U " files have been generated and GENLOG and COMNAV run. For this reason the manual processing approach was taken and the problem solved by setting the elevation mask lower in the batch input file. The " batch processing " routine was now used to ensure the use of the corrected input file parameters.

4. Using Precise Ephemerises.

Processing the Ashtech data using the precise ephemeris file provided by DMA, involves exactly the same procedure as for the broadcast ephemerises. The only step in the procedure which differs is in the MAKEUFIL program where the user determines which kind of ephemeris file, broadcast or precise, will be used to make the undifferenced data file for processing.

5. Results of Ashtech processing.

The problems encountered with day 340 data are not solvable by the vendor's software at the present time. Kinematic GPS surveying is still in its initial phase and

much work is being done to overcome the many problems and idiosyncrasies involved in the data processing. The day 341 processing furnished a large trajectory file of some 20,000 points covering five and a half hours of data collection. This file overlapped entirely with the reference trajectory providing almost three hours of processed and reference trajectories for comparison.

C. TRIMBLE DATA PROCESSING.

1. Method

Processing the Trimble data involved the same basic steps as for the Ashtech data processing. The program was run on the data to determine satellite tracking times and periods of simultaneous satellites per site and per session. The times when reference measurements were taken to the Trimble antenna on the ship, from the dock, prior to and after sailing, were noted for initialization. The downloading of the Trimble data produces four files :

A data file (.DAT).

An ephemeris file (.EPH).

A message file (.MES).

A file with ionospheric correction parameters (.ION).

Figure 19 shows a flow chart of the Trimble processing procedure.

All Trimble programs begin by reading in the data from the .DAT - file. The program TRIMKIN was run to produce a batch - file for batch mode processing. TRIMKIN is an interactive program allowing the user to input information such as site positions, initialization and processing start and stop times and processing mode (continuous kinematic, stop and go kinematic etc.). Once the batch - file was produced it was edited and certain changes were made. The edited batch - file included parameters such as begin and end read - in times, compatible with the desired initialization technique and reference trajectory.

Initialization was set over one second (at one specific epoch) according to the measurement time of the reference point used for initialization. The baseline between the fixed station and the Trimble antenna, at the instant of initialization, was calculated in Earth - Centered Earth - Fixed coordinates and the parameters dx, dy and dz were edited into the batch - file. The constraints on the initialization and integer ambiguity resolution for the Trimble programs demand that the " integers " be less than .25 off an integer number. At least four satellites have to have good integer values for the processing

to begin. A parameter was entered into the batch - file to indicate that the initialization process be iterated until the above condition is met. A path and name for the ASCII output trajectory file concluded the preparation of the batch - file.

The program TRIMMBP was run in the batch mode, using the prepared batch - file. When the data is read in to the program a " pre- processed " scratch file is produced which is about twice the size of the two data files. This scratch file is comparible to the Ashtech " U " files and contains the undifferenced phase data needed for the processing. Once the initialization was completed the solutions for the sequential positions were written to the ASCII trajectory file as the screen scrolled through the various iterations of processing at each point to indicate that the program was running. The above procedure was applied to both days 340 and 341 data sets and the programs ran without any apparent problems.

2. Results

Each data set produced an output file of about 1.2 MB. These output files contain dx, dy and dz values relative to the fixed reference station. A program furnished by the vendor, with the fixed station coordinates edited into the output file, converts this output to latitude, longitude and height coordinates. This program was run on both data sets. Both data sets included full overlap with the reference trajectories and provided once again almost three hours of processed and reference trajectories for comparison.

Table 6. ASHTECH AND TRIMBLE SOFTWARE COMPARISON.

	Similarities		Differences
1	Both use the undifferenced, preprocessed files for data processing	1	Ashtech makes "U" files as a separate step; Trimble creates the scratch file as a preliminary step
2	Both can run in automatic, batch or manual modes	2	Ashtech processing is quicker and outputs a large results file. Trimble processing is slower but outputs a smaller results file
3	Both can take either precise or broadcast ephemerises	3	Formats of the precise ephemeris files differ and they are introduced at different stages in the programs.

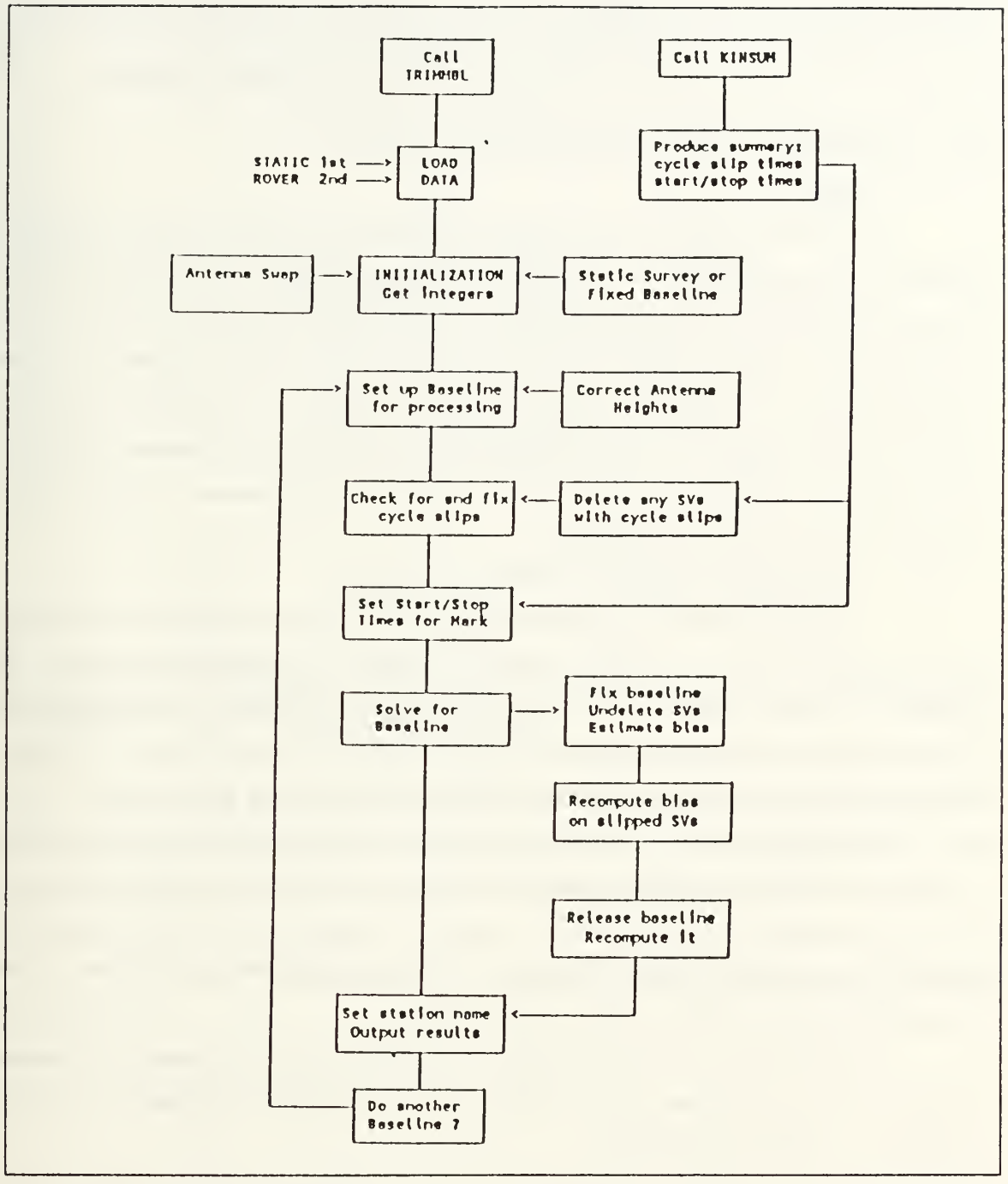

Figure 19. Trimble data processing procedure.

Ashtech " U " files are saved once produced and therefore can be re-used. Trimble scratch files are produced each time processing begins and are deleted at the end of the

processing procedure. The Ashtech output file contains position vectors from the reference station as well as the calculated coordinates of the moving receiver. The Trimble output file contains only the dx, dy and dz offsets from the reference receiver.

D. PROCESSING WITH NGS SOFTWARE - OMNI.

1. Method

Since NGS uses data from various sources, they have developed a software package for processing GPS data collected by different receivers. The software package, called OMNI, has a four step procedure for processing GPS data. First the data is converted from the receiver format to the NGS format, ARGO. Next the data to be used in the processing is merged into a single data base. At this point any editing of data can be done, either prior to actually processing the data or after the processing step. Lastly the programs actually computing the desired output are implemented. Figure 20 illustrates the four step OMNI procedure.

2. Processing GPS Data With OMNI.

The program OMNI.BAT provides a main menu from which the various processing steps and subroutines can be called. The options in the main menu include : Reformat Raw Data; Merge a Data Base; Data Editing; Static Solutions; Kinematic Solutions; Rapid Static Solutions and Utility Programs. Figure 21 shows a flowchart of the processing procedure undertaken, including the input and output files.

Once the raw data has been reformatted and the data base merged, a number of files are available for plotting. These files assist the user in determining where cycle slips have occurred and where editing is necessary. Editing can either be done manually or automatically by the program. Once the user is satisfied that the data base is ready for processing, the required solution mode is called.

The kinematic solutions program allows for a two step approach to processing the data. First the program makes a quick pass over the data searching for cycle slips. This is done using two algorithms :

The range differences utilizes the fact that phase biased range is affected by cycle slips while pseudo - range is not. The range differences are defined by :

$$\frac{c}{f}\left[(P_\alpha^i)_t - (P_\alpha^i)_{t_0}\right] \; - \; \left[(R_\alpha^i)_t - (R_\alpha^i)_{t_0}\right]$$

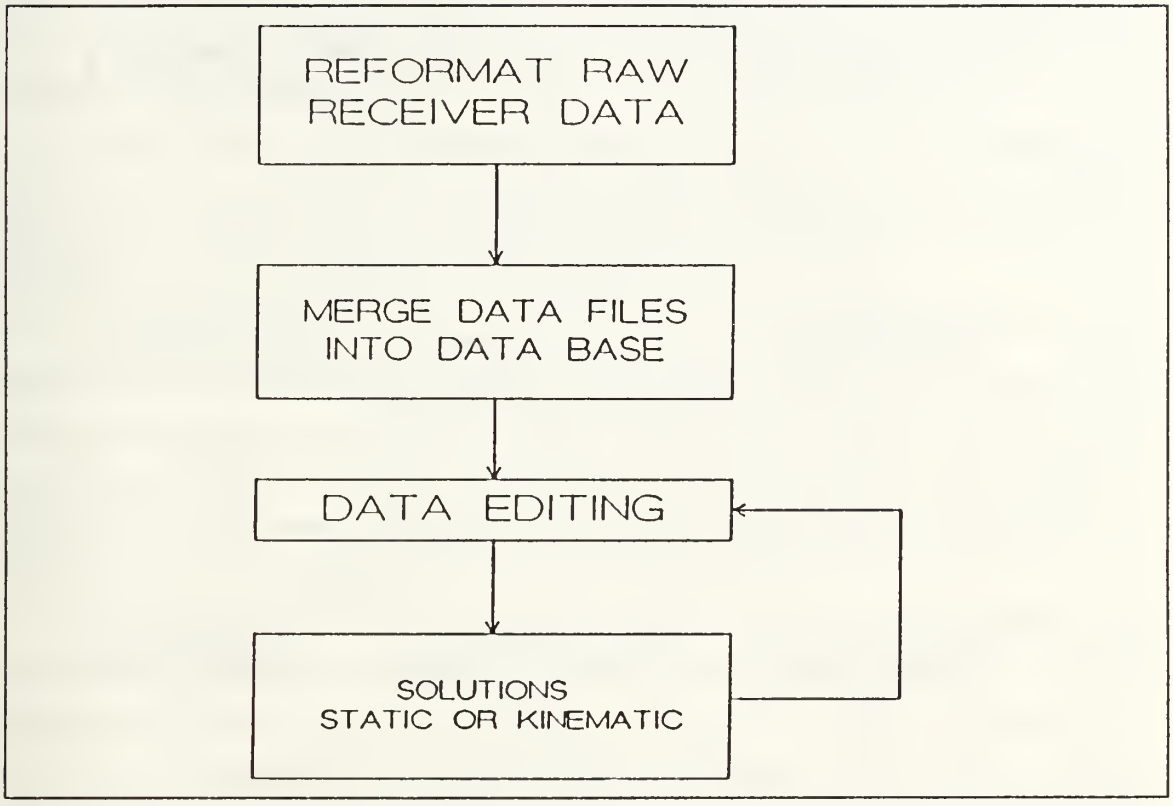

Figure 20. · NGS - OMNI Processing Steps.

where P is the phase biased ranged and R is the pseudo - range, α is the station and i is the satellite. This technique, though limited by the noise in the pseudo - range measurements, is effective in finding cycle slips on both L_1 and L_2 frequencies.

The ionosphere residual is much more sensitive than the range differences and is defined by :

$$(P_\alpha^i)_{L_1} - \frac{f_1}{f_2}(P_\alpha^i)_{L_2}$$

Since this is a linear combination of the L_1 and L_2 phases, determining the frequency on which the cycle slip has occurred is not a simple matter. [Ref. 8 : p. 7 - 4]

The results of these calculations can be plotted for the user to determine correction procedures. The next step is to solve for the mobile receiver's instantaneous po-

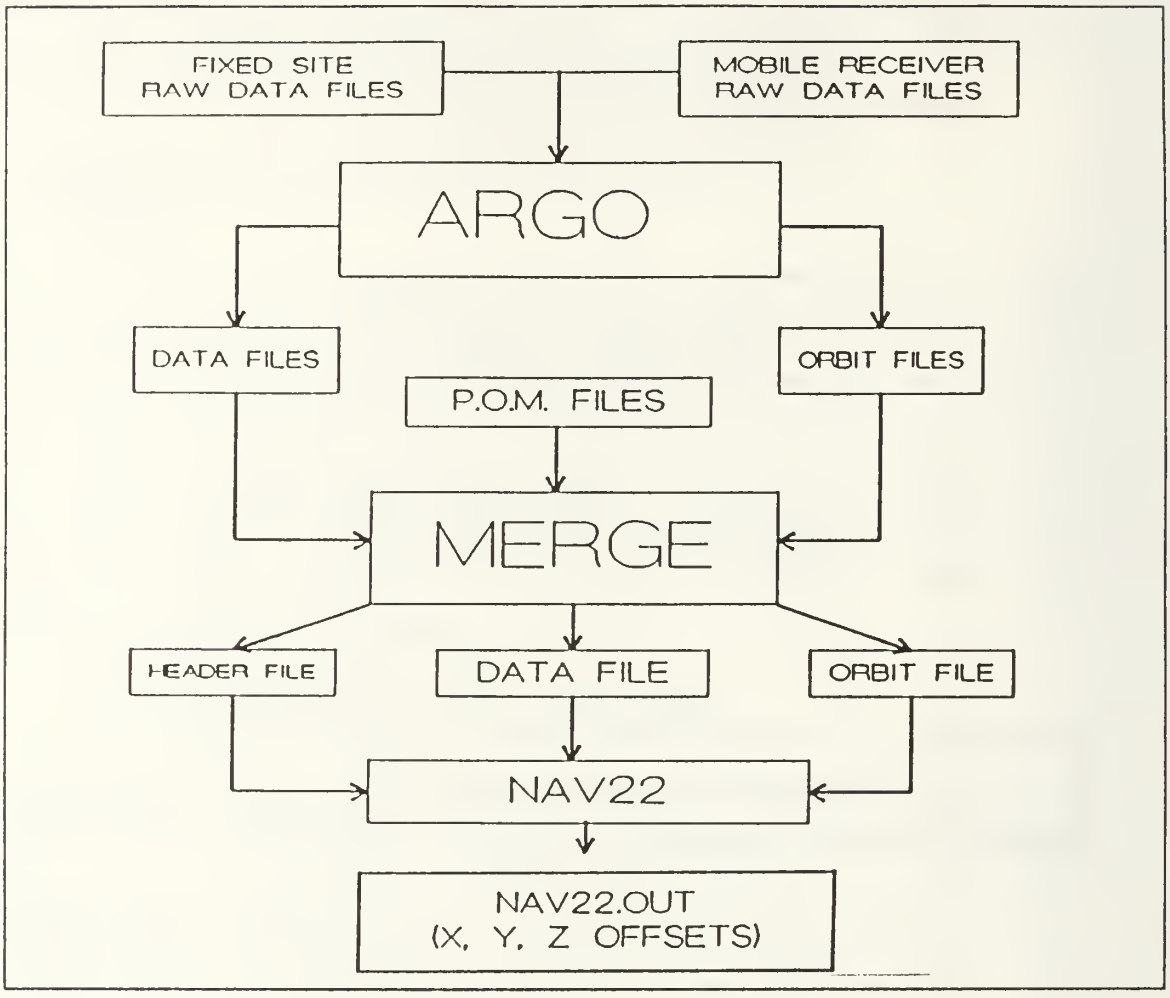

Figure 21. OMNI Processing Flowchart.

sitions. Initialization for integer cycle ambiguity resolution is implemented on a pre - calculated fixed baseline at a single epoch. As with the Ashtech and Trimble software, the main observables used in this program are the double differenced phase measurements. The results of this program are written to an output file in the form of offsets, in earth - centered earth - fixed coordinates, from the mobile receiver's original position. These offsets are readily converted to latitude, longitude and height coordinates compatible with the reference trajectories.

3. Results.

The output of the NGS software resembles the Trimble software output in that it contains offsets from a certain position. This was utilized to convert the trajectory

output to the reference trajectory format. A program was written to reformat the OMNI output to the Trimble output format whereafter the same procedure was taken as for the Trimble trajectory files.

In the " Merge " stage of the processing, using the Trimble data, a message indicating a lack of orbit data for satellites 9 and 11 appeared. This occurred on both days, 340 and 341 and using Broadcast and Precise ephemerises. In figure 10 it is evident that around 08:00 UTC, satellite 11 set and satellite 9 rose.

The processing program terminated on both days just before 08:00 UTC with a message that too few satellites were available. This occurred due to the fact that satellite 9 was not used as it should have been. The Precise Ephemeris, when used, is used only for the satellite positions. The satellite clock data is obtained from the Broadcast Ephemeris. Since the orbit data for satellite 9 was missing, using the Precise Ephemeris did not overcome the problem.

In order to overcome this problem, a TI 4100 data set for the same session was converted to ARGO. The orbit file thus produced contained ephemeris data for satellite 9. This orbit file was added to the Trimble orbit file and the " Merge " program re-run. Once the data was successfully merged into a new data base, the processing program was re-initialized. The resulting output file contained a trajectory for the full processing period, until 10:00 am UTC when the reference trajectory ended. This enabled a comparison of the Trimble trajectory for day 341 using the two processing programs, TRIMMBP and OMNI.

E. REFERENCE TRAJECTORY DATA PROCESSING.

1. General.

The reference trajectories for the Ashtech and Trimble antennas on the ship were derived from the Krupp Atlas Laser Systems output combined with the two-axis and heading gyro output from the ship. The Polarfix laser system provided a trajectory of the laser reflector which was mounted on the mast of the ship along with the GPS antennas, while the gyro systems provided the orientation of the ship. This was needed to connect the laser mirror trajectory to the GPS antenna locations in an ECEF coordinate frame.

The Polarfix laser system was set up at the Monterey Bay Aquarium Research Institute (MBARI), about five kilometers from the area of operations where the ship was tracked. Data for this system was recorded on a PC with the National Institute of Standards and Timing (NIST) Automated Computer Timing System (ACTS) used for

calibration. Time tags for each laser range and angle measurement were to within 50 ms of UTC. Data were collected at a rate of three measurements per second. The noise in the laser measurements was on the order of 1 m in the range and 0.01 degrees in the angle. This was reduced to about 60 cm in the post- processing of the reference data with a Hamming window used to average the data to one second intervals.

Seven small laser reflectors were placed on the ship, with one below the Ashtech and one below the Trimble antenna. These were used in the development of the ship model with time tagged laser measurements taken to each reflector while the ship was tied up to the pier. Each measurement was repeated five times with a least squares adjustment used to compute the positions. This resulted in a measurement accuracy on the order of one centimeter. A wrist watch was used to time the laser shots from the pier. This watch was then calibrated by comparing it to a standard, accurate to within one second of UTC. The two-axis vertical gyro and the heading gyro were calibrated by implementing the measurements of the antennas pre-and post-mission. The gyro measurements were time tagged and compared to the measured alignment of the Ashtech and Trimble antennas.

The Polarfix laser system produces only horizontal (two dimensional) positions. NOAA Geoid and Tide models were used with the vertical two-axis gyro data to produce the height component. The angles measured were used to move the trajectory of the laser reflector, across the two meter lever arm, to the GPS antennas. More details on the reference trajectories and the method used to compute them, can be found in the ION GPS - 91 Technical Program Proceedings [Ref. 9].

Four reference trajectory files were produced - one for each receiver (Ashtech and Trimble) per session. These files included times in seconds of day (sod) and latitude, longitude and height. Each file had about three hours of data with two gaps (of about 5 minutes or 300 data points each) when the laser systems broke off to re- calibrate. Other small gaps, ranging between one and 20 seconds occurred from time to time when the laser measurements or gyro system outputs did not meet specified criteria. This created reference data files which were of a different format to both the Ashtech and Trimble trajectory files as well as having some gaps of various sizes.

F. DATA EVALUATION PROGRAMS.

With three different kinds of trajectory files, each in a different format, it was necessary to convert all the data to a standard format. Since the reference trajectory output was in a format which could readily be converted to distances, it followed that this would

be the standard format. Two programs were written, one to convert the Ashtech trajectory files and one to convert the Trimble trajectory files to the reference trajectory files format. The WGS - 84 ellipsoid was used to calculate that one degree of latitude at the fixed station (Beach - Lab) is equal to 111.1875 km while one degree of longitude is equal to 89.4737 km.

Having converted all the trajectories to the standard format with four fields - one of time in seconds of day, two in degrees and decimal degrees carried out to seven places and one in meters and decimal meters carried out to two decimal places - it was possible to run a differencing program to evaluate the results. DIFDAT3 is a program which reads two files, prompts for the relevant fields in each file to difference and outputs a results file. The differencing is done at the points in the second file read in which match the first file in the time field. There was no interpolation of data to difference between the times of the reference trajectory output; the points differenced were the existing points in the reference file only. Lastly, by multiplying the differences obtained by the relevant scale factor, the offsets between the trajectories were converted from degrees to distances in units of centimeters.

A program was written to edit out the points where differences greater than 3σ appeared. For the Ashtech output this meant that all points with differences of over two meters in any direction, latitude, longitude or ellipsoidal height, were considered as outliers. Most of these points occurred when the reference trajectory resumed after a gap in the data set, indicating a systematic problem in the end points of the interpolation algorithm used to produce the reference data file. For the Trimble output, 3σ was about 10 m, due mainly to the trend at the end of the session when the GPS solution deviated from the reference trajectory. In most cases the number of outliers did not exceed 2 % of the entire data sets. The outliers were eliminated from the reference trajectory and the differencing program rerun. The statistical information output by the DIFDAT3 program excludes the outliers.

The output files, containing the differences between the latitude, longitude and height of the GPS and reference trajectories, were then plotted. Plots were made of distance between the trajectories in each of the coordinate directions as a function of time. Examples of these plots for both Ashtech and Trimble data are shown in the next chapter which discusses the results obtained.

All GPS trajectories and reference trajectories obtained in this experiment contained random noise. In order to evaluate the noise levels in the GPS trajectories it was necessary to reduce, as much as possible, the noise in the reference trajectories. For this pur-

pose a program was written to filter the reference data using a weighting algorithm. The von Hann (raised cosine) window used is essentially a low pass filter which reduces the high frequency noise prevalent in the data. [Ref. 10] A 120 second window was used to average the data with 60 second steps used. This resulted in smoothed output at a one point per minute rate. The GPS trajectories determined were also filtered with this von Hann window to analyze the low frequency trends in the output.

VI. RESULTS

A. GENERAL

The results include the output of the various kinematic solutions and auxillary programs. The processing procedures were described in the previous chapter. This chapter also includes statistical analysis of the processed data. The trajectories obtained from the GPS measurements were compared to the reference trajectories and the difference between the two were converted to distances in three directions, latitude, longitude and ellipsoidal height. These differences were then plotted as a function of time to examine trends.

In order to determine the magnitude of the various effects on the GPS measurements, it was necessary to process the data a number of times, each time changing certain parameters in the input. The following factors were considered as candidates for affecting the accuracy of a GPS kinematic survey :

Applying / ignoring the tropospheric correction; Initialization; Satellite geometry - DOP factors; Broadcast / Precise Ephemeris; Data interval and Multipath. Since both the Ashtech and the Trimble receivers used are dual channel, the ionospheric correction was not considered. It was assumed that this factor was eliminated by direct measurement.

B. SOLUTION ACCURACY

Many problems were encountered during the processing stage. Each software package has its advantages and limitations with some options easily applied and others needing more complicated input. Kinematic GPS processing is continually undergoing improvements as attempts are made to overcome the many idiosyncracies inherent in the solution. Table 7 provides a concise summary of processing.

1. Ashtech Day 340 Data

Both GPPS and OMNI successfully initialized on the baseline input for day 340. However, both programs terminated prematurely indicating that too few satellites were available to continue processing. Examination of the raw data files at the time the program terminated revealed a seven second gap in the ship data. Neither the GPPS nor the OMNI programs could overcome this gap which produced a cycle slip problem. OMNI has an interpolation algorithm for automatically fixing such cycle slips. However, it did not succeed in bridging this seven second gap. Furthermore, attempts to re-

Table 7. **DATA PROCESSING:** Pluses indicate data processing which overlapped with the reference trajectory.

Data	Software	Day 340 Broadcast	Day 340 Precise	Day 341 Broadcast	Day 341 Precise
Ashtech	GPPS			+	+
	OMNI			+	+
Trimble	TRIMMBP	+	+	+	+
	OMNI	+		+	+

initialize the kinematic survey, while the ship was underway, using a receiver coordinate obtained from the Trimble trajectory, were unsuccessful. The programs rejected the cycle ambiguities calculated for this instantaneous baseline as unreliable.

2. Ashtech Day 341 Data

This data was successfully processed with the Ashtech software using both Broadcast and Precise Ephemerises. A trajectory of over five and a half hours was obtained in each case, including almost three hours of overlap with the reference trajectory. Options such as including or excluding the tropospheric correction and fixed integer or floating real cycle ambiguities, were readily implemented.

The plots of the comparison of the Ashtech trajectory processed with GPPS, with the reference trajectory, are shown in figure 22. These plots show the unfiltered output at a one second recording and processing rate. Plots of the same difference output as in figure 22, filtered using a 120 second von Hann window, stepped every 60 seconds, are given in figure 23.

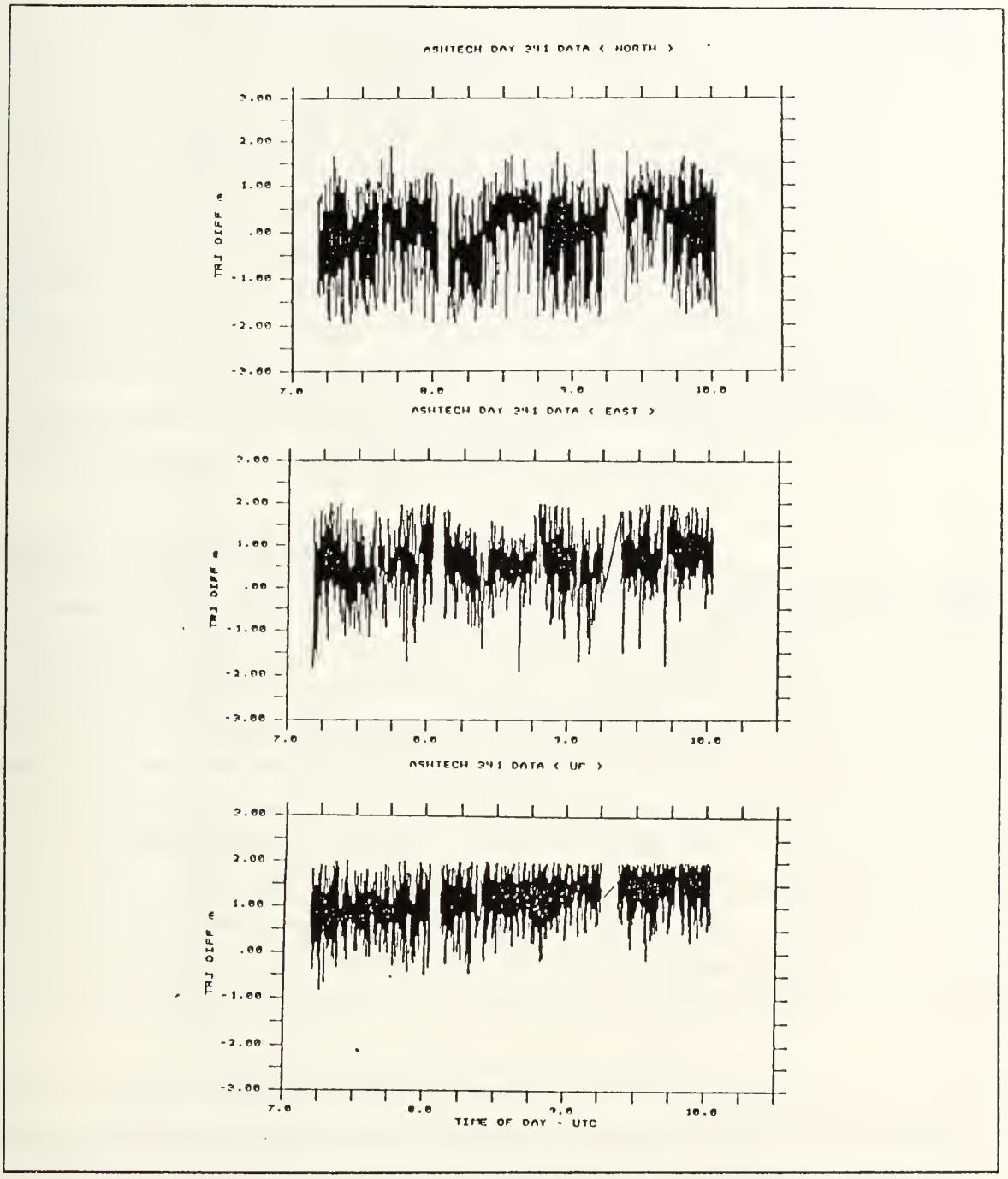

Figure 22. Ashtech Day 341 - Reference: Differencing output between Ashtech day 341 and reference trajectory - $\Delta\phi, \Delta\lambda$ and Δh components in meters. Broadcast Ephemeris used.

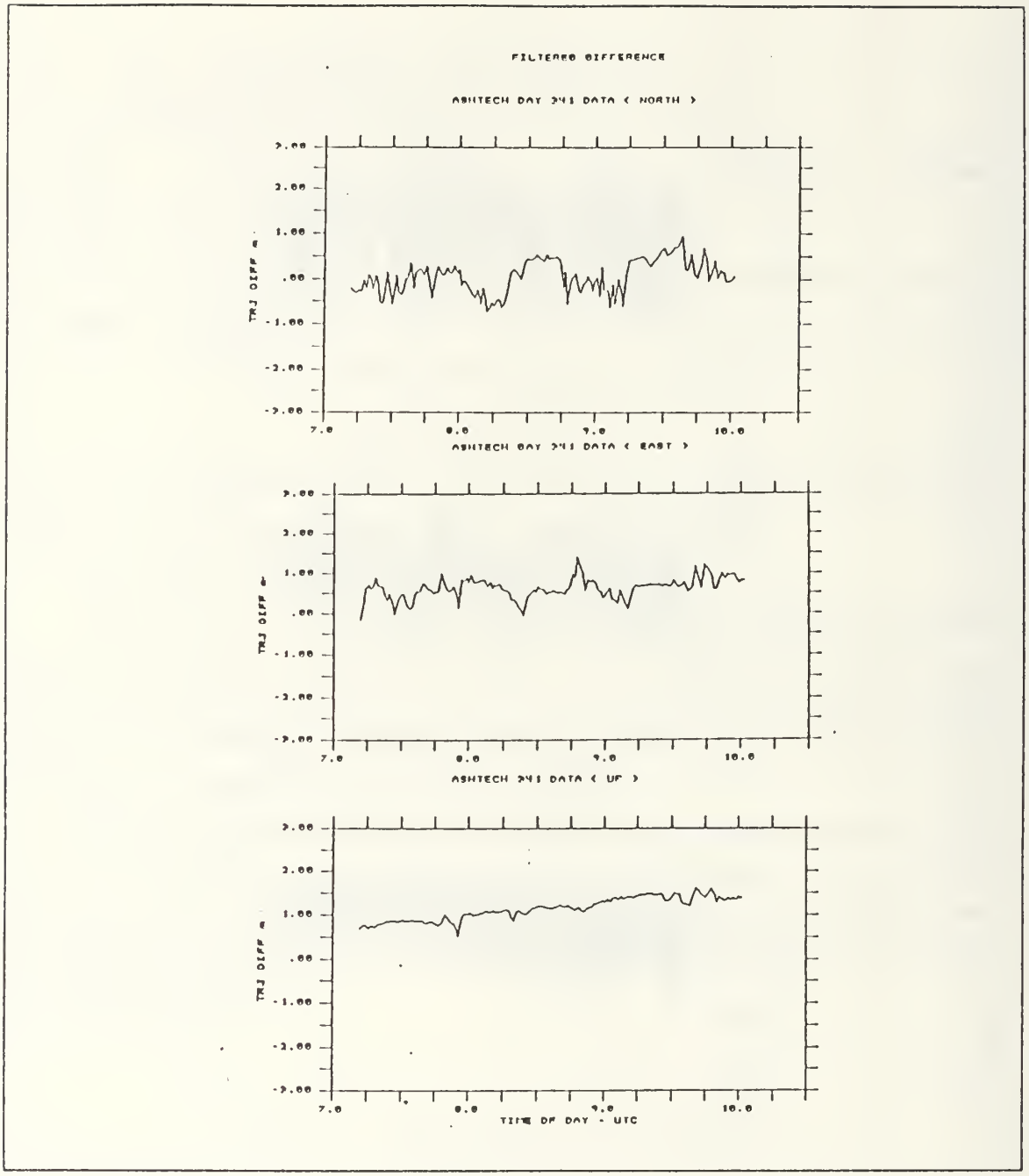

Figure 23. Ashtech Day 341 - Reference (Filtered): Differencing output between Ashtech day 341 and reference trajectory - $\Delta\phi$, $\Delta\lambda$ and Δh components in meters. Broadcast Ephemeris used. Data filtered with 120 second window.

The noise in the unfiltered plots is on the order of 50 cm in each direction. Considering the fact that the noise in the reference trajectory, determined at calibration, was on the order of 60 cm, it would appear that the Ashtech trajectory is very close to the " truth ". There is no bias in the latitude, about a 50 cm bias in the longitude and a one meter bias in the height. This bias in the height will be addressed later.

Processing the Ashtech day 341 data , using OMNI, produced a two hour trajectory which terminates soon after the reference trajectory begins. This occurs just before 07:30 UTC on day 341. The program indicated that it could not continue processing due to too few satellites being available. This appears to occur when satellites 11, 16 and 18 are setting and before 9, 13 and 15 rise. The first ephemeris update for satellites 9 and 13 is obtained at 09:00 UTC, which could be causing the problem. Table 8 provides a summary of the Broadcast Ephemeris updates for day 341.

Table 8. DAY 341 BROADCAST EPHEMERIS UPDATES

SV	04:51	06:00	07:00	08:00	08:51	09:00	09:51	10:00	11:00	12:00
2		+		+	+					+
6			+	+	+	+	+			+
9						+	+	+	+	
11			+	+		+		+		
13						+		+	+	
14	+									
15				+				+		+
16		+	+	+	+					
18	+			+						
19	+			+						

Table 9 shows the comparison of the Ashtech GPPS and NGS OMNI output files for the nearly two hours of overlap. Here the data was processed with a five second interval. The initialization of the kinematic solution was identical, using the same baseline input for both programs. Since there was not enough overlap with the reference trajectory, it was decided to compare the OMNI and Ashtech solutions.

It is evident from this table that, other than a bias on the order of one meter in the height, the Ashtech and OMNI trajectories are within a few centimeters of each other. Examination of the trajectories produced by each program, reveals that the

Table 9. OMNI - ASHTECH COMPARISON

PROGRAMS	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*	NO. OF POINTS
OMNI - GPPS	1 ± 5	4 ± 1	103 ± 13	1250

Ashtech software produced a height, about one meter above the initialization value used, right from the first epoch. This bias appears to be consistent over the two hours. Figure 24 shows a plot of the height component for the two trajectories, GPPS and OMNI, as a function of time.

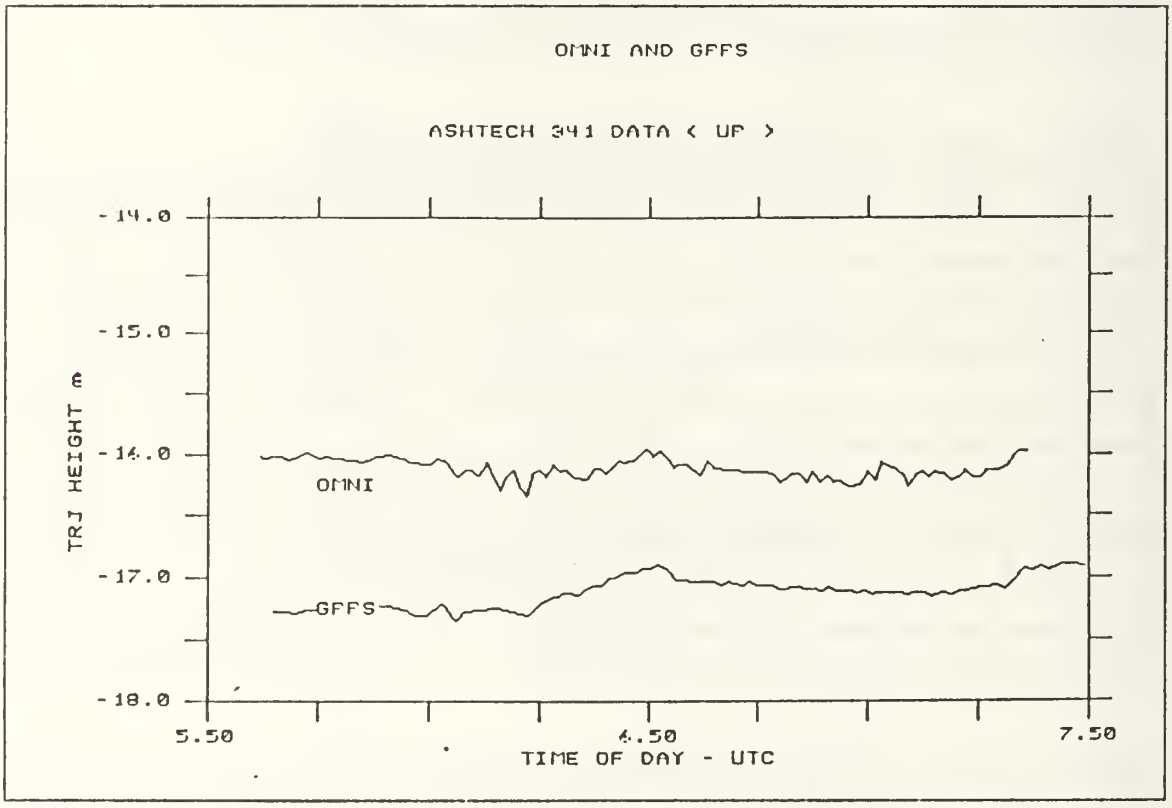

Figure 24. Ashtech GPPS and OMNI Height Comparison: A comparison of the height output by the Ashtech software and OMNI. Ashtech day 341 data used in both cases, with trajectories filtered with 120 second window.

The reference trajectory produces a height of -17.8 m from the time it begins at about 07:10 am. UTC. This is about one meter below the Ashtech height at that time and two meters below the OMNI height.

3. Trimble Days 340 and 341 data

Processing with the TRIMMBP program, using a batch file input, produced a trajectory which overlapped fully with the reference trajectory. Certain options, such as choosing the ephemeris file or data processing interval, are easily implemented in the batch file. However, other options, such as not using the tropospheric correction or rounding the cycle ambiguity to a fixed integer, involve complex changes in the batch file. The batch file loops in and out of various programs and is not a " user friendly " approach to the processing. None of the attempts made to process data with the TRIMMBP program, implementing the options just mentioned, were successful.

Figures 25 and 26 show the unfiltered output of the Trimble data for days 340 and 341, differenced with the reference trajectory. These Trimble trajectories were obtained using the Broadcast Ephemeris. Comparisons between the Trimble Broadcast and Precise Ephemeris trajectories can be seen in the section on ephemeris types. Figures 27 and 28 show the plots presented in figures 25 and 26 respectively, filtered with a 120 second von Hann window.

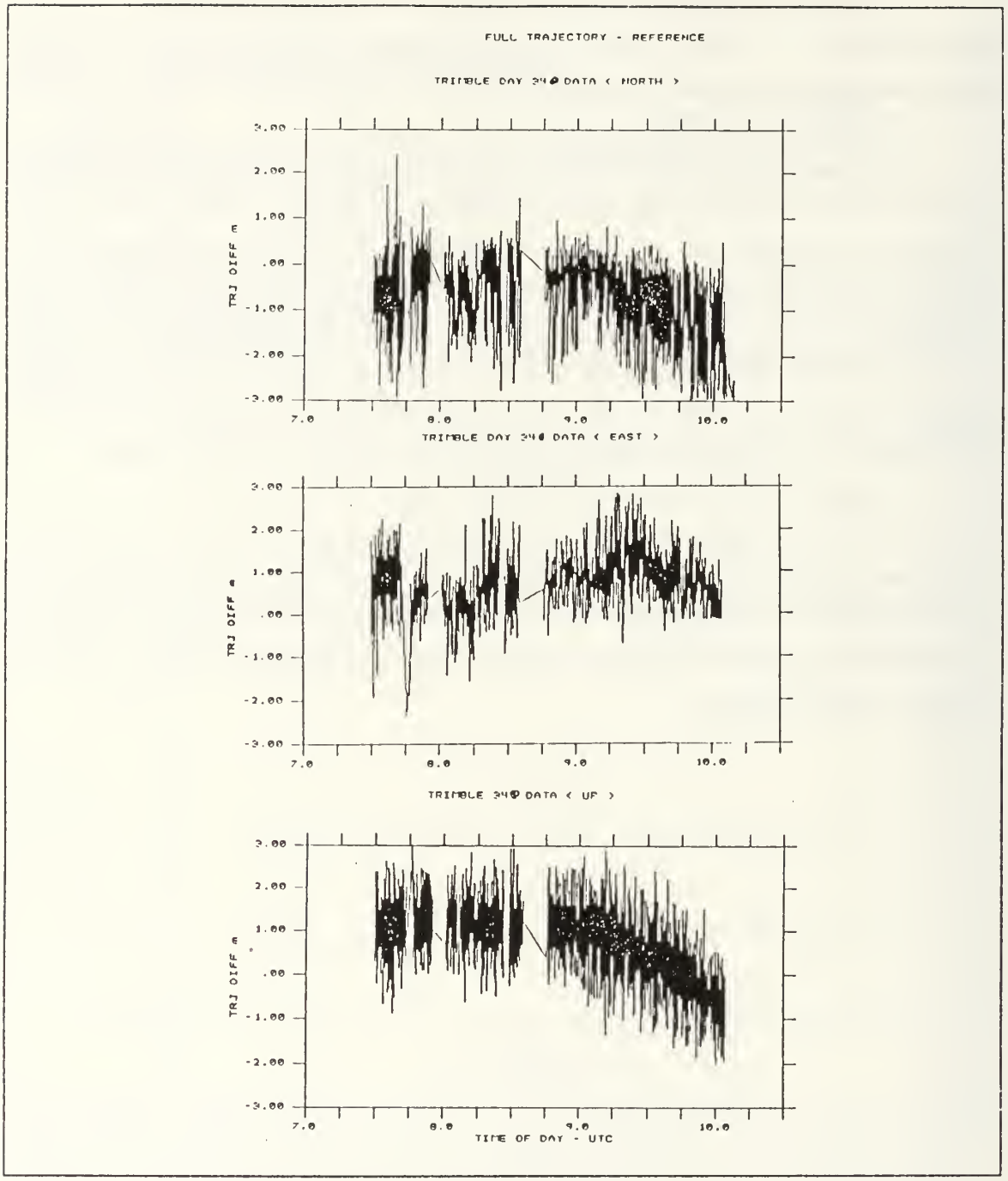

Figure 25. Trimble Day 340 - Reference: Trimble Day 340 data differenced with reference trajectory - $\Delta\phi$, $\Delta\lambda$ and Δh components in meters.

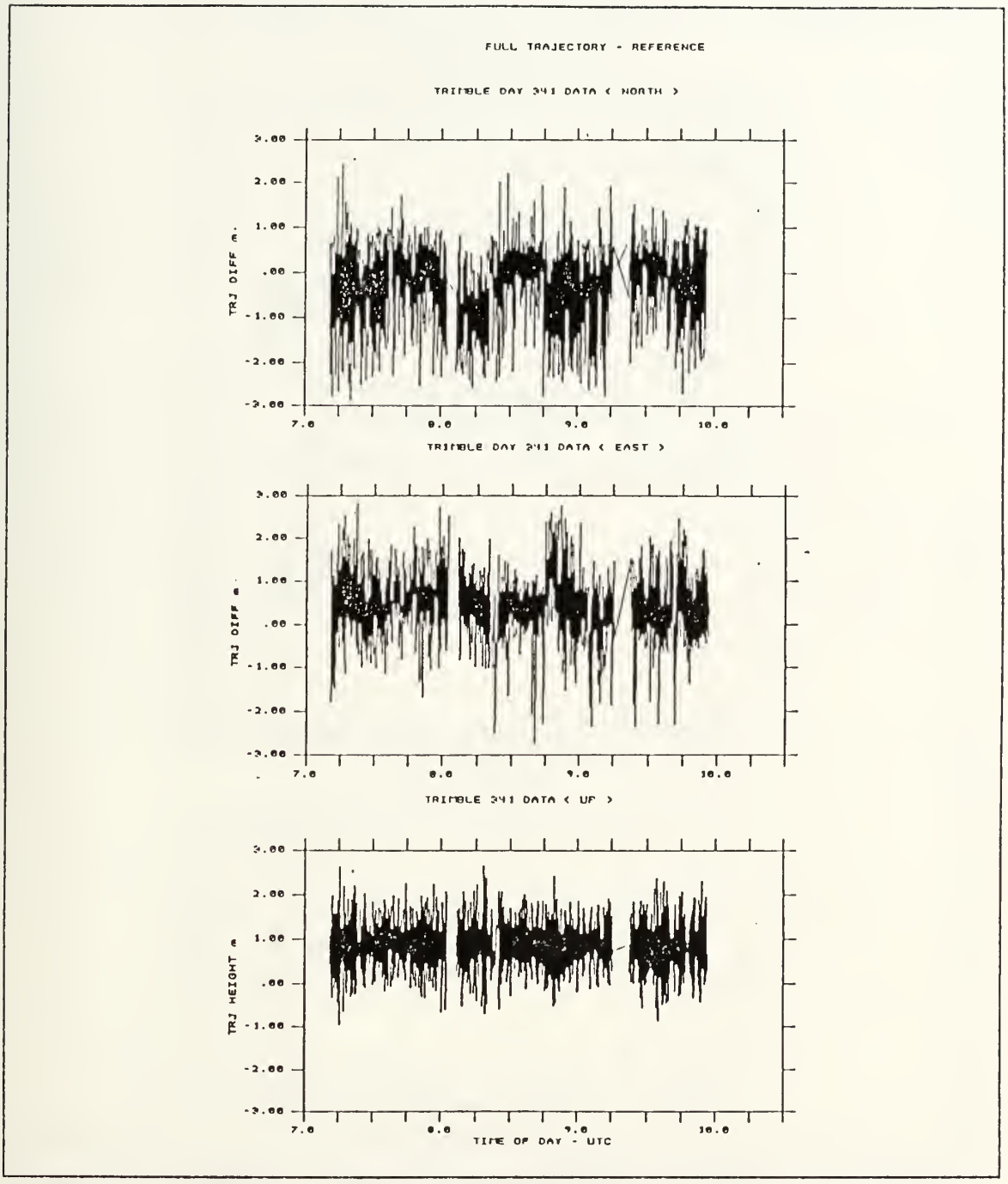

Figure 26. Trimble Day 341 - Reference: Trimble Day 341 data differenced with reference trajectory - $\Delta\phi$, $\Delta\lambda$ and Δh components in meters.

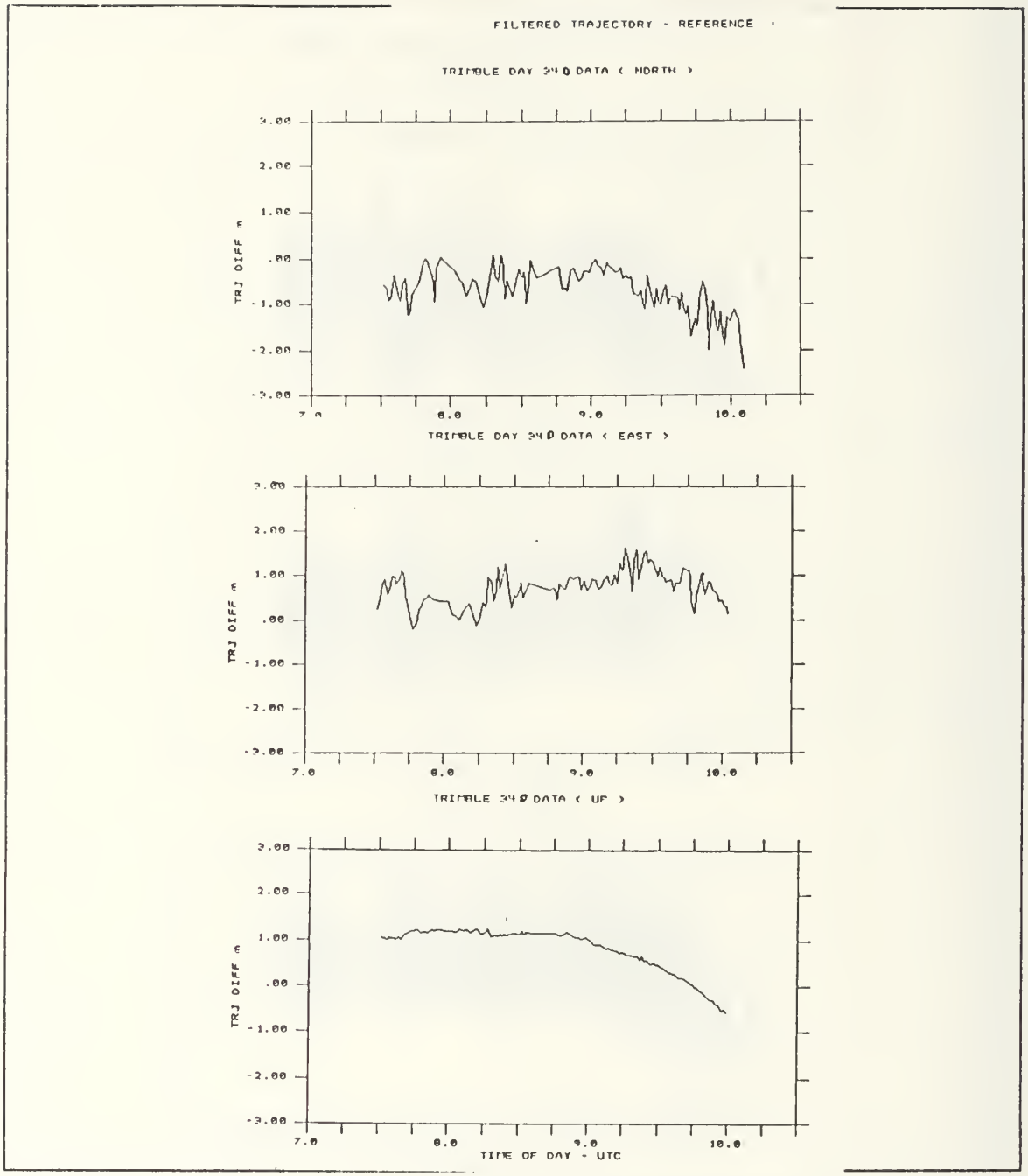

Figure 27. Trimble Day 340 - Reference (Filtered): Trimble Day 340 data differenced with reference trajectory - $\Delta\phi$, $\Delta\lambda$ and Δh components in meters.

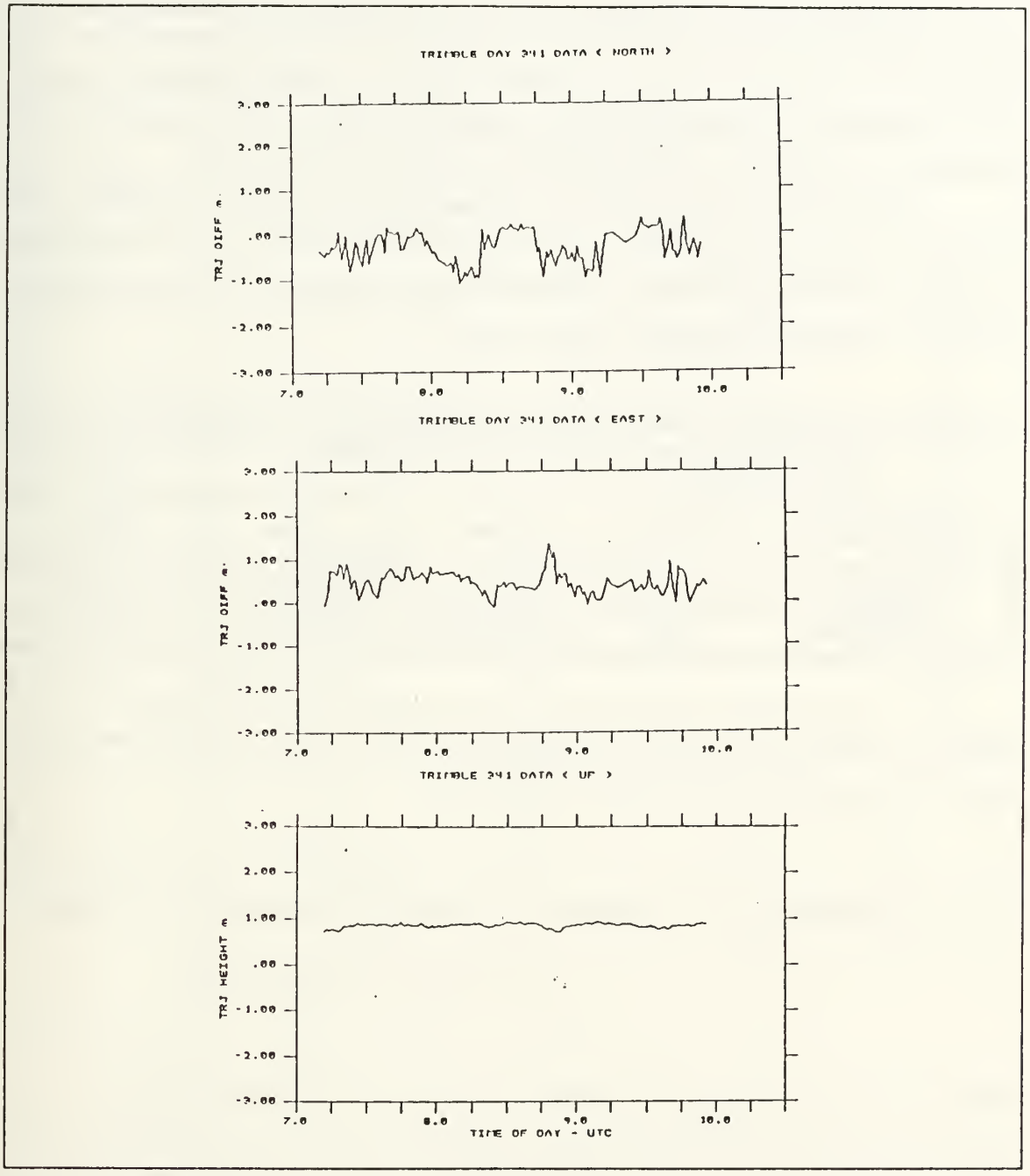

Figure 28. Trimble Day 341 - Reference (Filtered): Trimble Day 341 data differenced with reference trajectory - $\Delta\phi$, $\Delta\lambda$ and Δh components in meters.

It appears from these plots that the Trimble solution is comparable to the Ashtech solution. The noise in the Trimble solution is only slightly greater than in the Ashtech solution, but once again it is on the order of the reference solution noise - 60 cm. The trends in the plots, in all three components, which seem to be related to the satellites used and the changing configuration, are similar in both cases.

The Trimble data for both days was successfully processed with OMNI. The output in each case was differenced with the TRIMMBP output. The results of these comparisons are given in table 10. The general trends apparent in the Trimble software output are apparent in the OMNI output as well.

Table 10. TRIMMBP AND OMNI COMPARISONS: A comparison of Trimble days 340 a 341 data, processed with TRIMMBP compared to OMNI, $\Delta\phi$, $\Delta\lambda$ and Δh differen in centimeters.

PROGRAMS	DAY	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*	NO. O POINT
OMNI - TRIMMBP	340	-0.1 ± 1	0.4 ± 1.4	2.8 ± 3	3850
	341	-0.2 ± 0.5	0.5 ± 1.3	3.2 ± 3.4	3850

Figure 29 illustrates the height component of the OMNI and TRIMMBP day 341 trajectories for the first two hours after initialization.

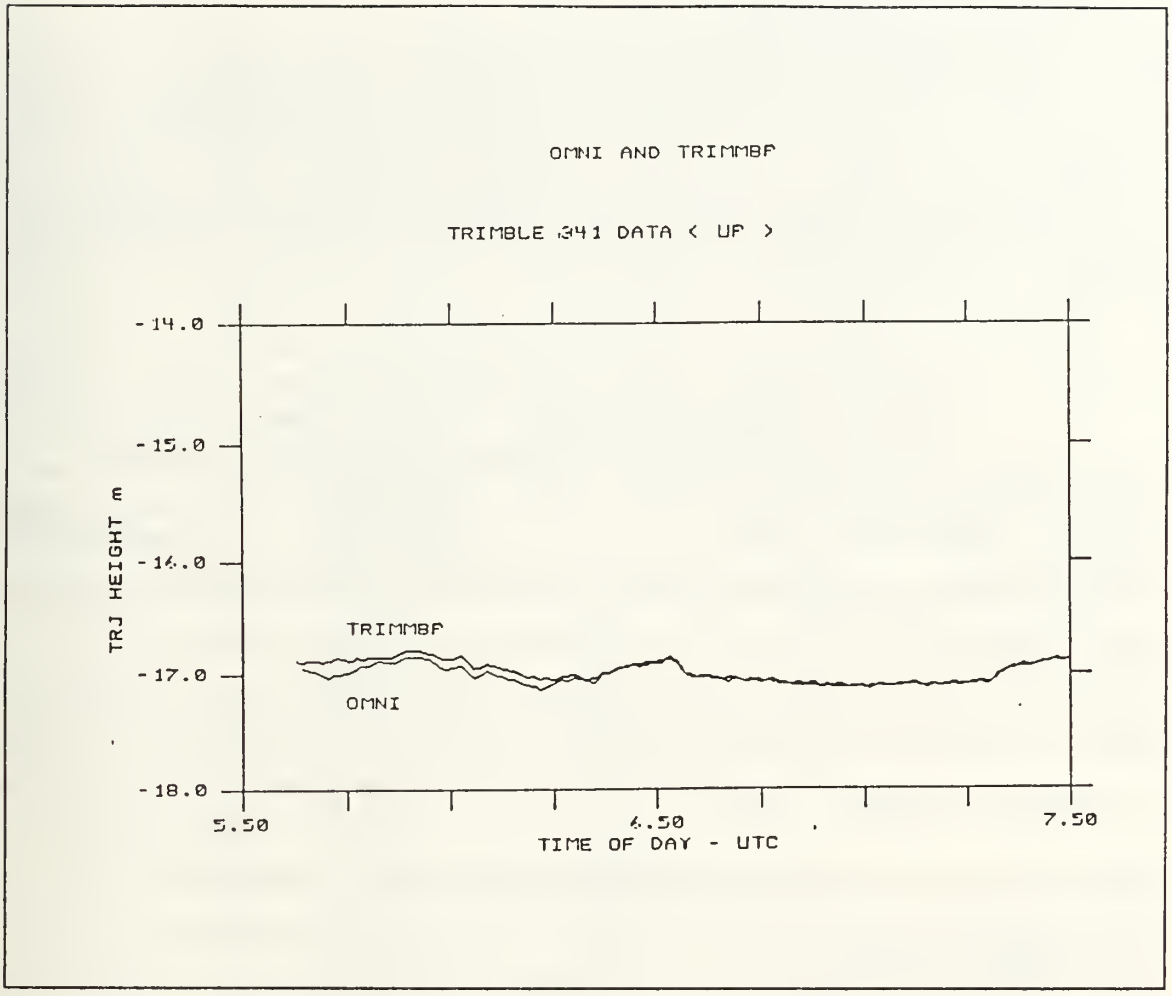

Figure 29. OMNI and TRIMMBP Day 341 Heights: A comparison of the first two hours of the height component output by OMNI and TRIMMBP using the Trimble day 341 data.

Comparing this plot to the equivalent plot of the Ashtech and OMNI heights for the same period, we can see that there is virtually no offset between the Trimble and the OMNI heights. Furthermore, the height produced by the Ashtech program is about the same as that produced by Trimble with both TRIMMBP and OMNI. The OMNI height using the Ashtech data is about one meter higher.

C. PARAMETER SENSITIVITY

1. Tropospheric correction.

The Ashtech day 341 data, processed with the Ashtech software, and the Trimble days 340 and 341 data, processed with the NGS software, were used for determining the effects of the tropospheric correction. Table 11 shows the results of the processing with and without the tropospheric correction, compared to the reference trajectory.

Table 11. TROPOSPHERIC CORRECTION COMPARISON: Results of processing with a without the tropospheric correction - average distance and standard deviation in ce meters from the reference trajectory in the ϕ, λ and h components.

DATA / PROGRAM	DAY	Δ	WITHOUT TROP. CORRECTION	WITH TROP. CORRECTION	NO. POI
ASHTECH / GPPS	341	$\Delta\phi$	1 ± 67	2 ± 67	85
		$\Delta\lambda$	58 ± 50	59 ± 50	
		Δh	111 ± 47	111 ± 47	
TRIMBLE / OMNI	340	$\Delta\phi$	-223 ± 81	-224 ± 81	81
		$\Delta\lambda$	149 ± 64	149 ± 64	
		Δh	456 ± 56	456 ± 66	
TRIMBLE / OMNI	341	$\Delta\phi$	89 ± 132	88 ± 131	857
		$\Delta\lambda$	615 ± 263	616 ± 263	
		Δh	604 ± 253	605 ± 252	

Table 12 shows a comparison of the trajectories output with and without the tropospheric correction. This table gives an indication of the magnitude of the differences between the pairs of trajectories. It does not indicate which trajectory is closer to the " truth ". For this reason the previous table, with each trajectory compared to the reference trajectory, was given. The difference in the number of points used to determine these statistics results only from the size of the files used. In the case of the Ashtech data, the full four and a half hours of data, from initialization at about 05:30 UTC, until the end of the reference trajectory at about 10:00 UTC, was used. For the Trimble data, only the time of overlap with the reference trajectory was used. However, the values did not change significantly for the Ashtech data when checked over the same period as the Trimble data.

Table 12. WITH TROPOSPHERIC CORRECTION - WITHOUT TROPOSPHERIC CORRECTION

DATA	DAY	SOFTWARE	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*	NO. OF POINTS
ASHTECH	341	ASHTECH	0.2 ± 0.8	0.2 ± 0.6	0.2 ± 1.3	15856
TRIMBLE	340	NGS - OMNI	-2 ± 2	-0.3 ± 0.5	-1 ± 1	6650
	341		-1 ± 8	0.2 ± 8	1 ± 7	8400

With a baseline less than ten kilometers in length, and both the reference receiver and the mobile receiver approximately at the same height, sea level, one would not expect significant changes due to the tropospheric correction. In theory the double differencing will eliminate this effect, which should apply equally to the two receivers.

The results verify that this is indeed true. The numbers, on the order of single centimeters, in table 12, are insignificant compared with the noise level of about 60 cm mentioned previously. For all intents and purposes there is no change in the solutions when applying or ignoring the tropospheric correction in this case.

2. Initialization

a. Fixed integer or floating real cycle ambiguity.

Again the Ashtech day 341 data and the Trimble days 340 and 341 data were used to compare the effect of rounding the cycle ambiguity to an integer. Table 13 shows the results of the processing using fixed integer and floating real values for the cycle ambiguities compared to the reference trajectory.

Table 14 shows the comparison between the various pairs of trajectories, using the floating real cycle ambiguity and the fixed integer value.

Most of the cycle ambiguities obtained from the baseline initialization were within 0.25 of an integer number. It is apparent from table 14 that the effects of rounding the floating real value obtained in each case are very small. In all the cases examined the changes were on the order of a few centimeters, certainly less than the L_1 wavelength of 19 cm.

Table 13. INTEGER AMBIGUITY COMPARISON: Results of processing using a fixed in and a floating real cycle ambiguity value - average distance and standard deviati centimeters from the reference trajectory in the ϕ, λ and h components.

DATA / PROGRAM	DAY	Δ	FIXED INTEGER	FLOATING REAL	N C PO
ASHTECH / GPPS	341	$\Delta\phi$	-19 ± 65	2 ± 67	
		$\Delta\lambda$	69 ± 50	59 ± 50	85
		Δh	96 ± 47	111 ± 47	
TRIMBLE / OMNI	340	$\Delta\phi$	-224 ± 81	-224 ± 81	
		$\Delta\lambda$	149 ± 64	144 ± 63	8
		Δh	456 ± 66	453 ± 66	
TRIMBLE / OMNI	341	$\Delta\phi$	88 ± 131	51 ± 130	
		$\Delta\lambda$	616 ± 263	549 ± 224	8
		Δh	605 ± 252	533 ± 239	

Table 14. FIXED INTEGER - FLOATING REAL AMBIGUITIES

DATA	DAY	SOFTWARE	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*	NO. POI
ASHTECH	341	ASHTECH	-17 ± 14	8 ± 12	-18 ± 28	158
TRIMBLE	340	NGS - OMNI	2 ± 1.5	4 ± 3	11 ± 7	665
	341		2 ± 6	7 ± 4	-5 ± 11	84

b. Initialization error

Inadvertently, in an initial processing attempt with the Trimble data, a mistake in the initialization was made. The initialization of the Trimble survey erroneously used a position about 2.0 meters off in the horizontal. The program accepted the initialization only when the constraint on the cycle ambiguity had been relaxed. In the initialization two of the five cycle ambiguities solved for had been closer to half a cycle off the integer. This provided a test of the sensitivity of the kinematic solution to the accuracy of the initialization.

Figures 30 and 31 present the plots of the Trimble solution, when the initialization was bad, differenced with the reference trajectory.

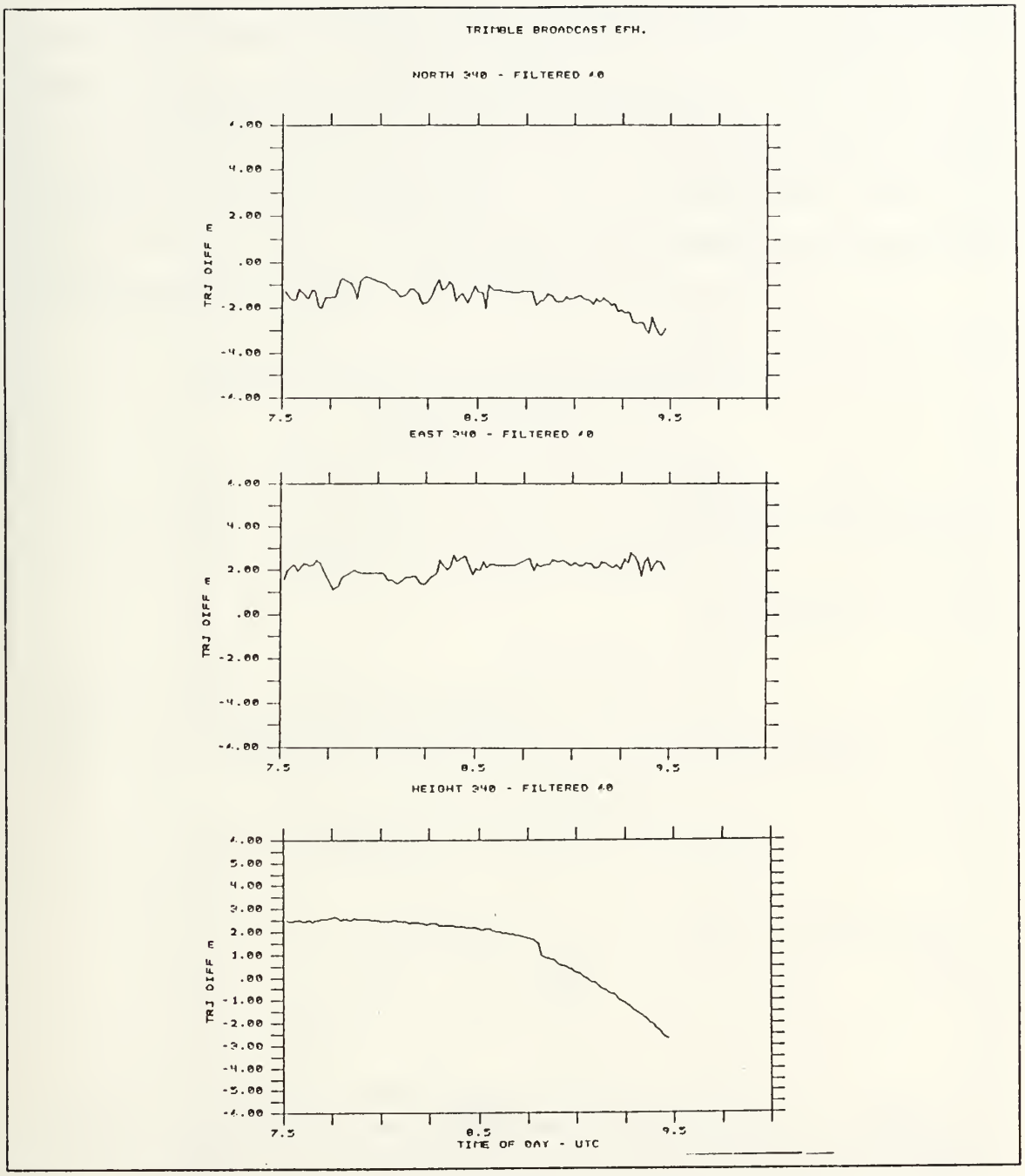

Figure 30. Initiaization Error Trimble Day 340 - Reference (Filtered): Trimble Day 340 data differenced with reference trajectory - $\Delta\phi, \Delta\lambda$ and Δh components in meters. BAD INITIALIZATION.

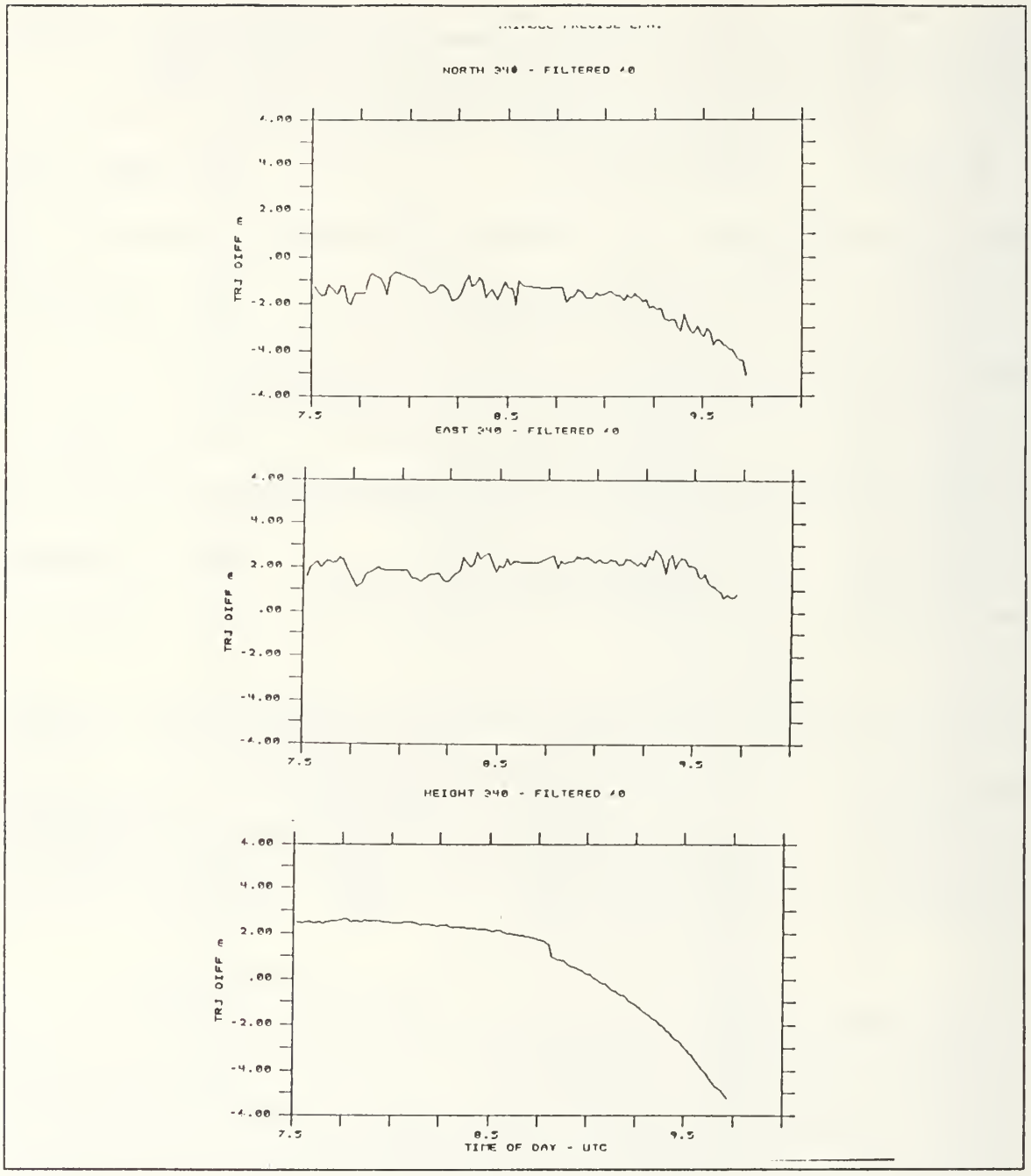

Figure 31. Initialization Error Trimble Day 341 - Reference (Filtered): Trimble Day 341 data differenced with reference trajectory - $\Delta\phi, \Delta\lambda$ and Δh components in meters. BAD INITIALIZATION.

It is evident from these figures that there is a problem with the Trimble trajectories. The increasing deviation from the reference trajectory after 09:00 UTC, especially in the latitude and height components, is a consistent trend. This consistency, even when some of the other factors examined were changed, indicates that the initialization may have been responsible for the deviations. The trend occurs using both TRIMMBP and OMNI and both Broadcast and Precise Ephemerises. Figure 32 presents the plot of the OMNI and TRIMMBP day 341 heights when the initialization of the Trimble antenna was bad.

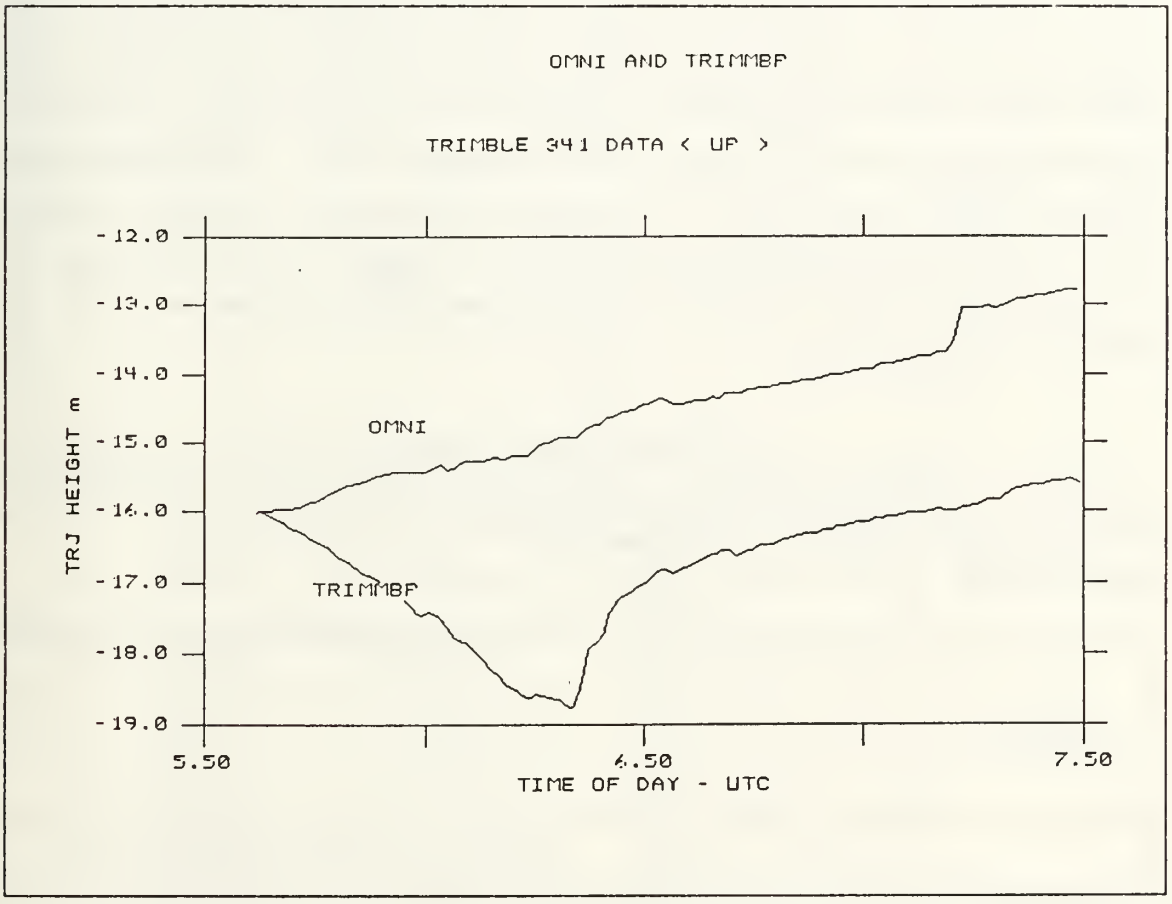

Figure 32. Initialization Error OMNI and TRIMMBP Day 341 Heights: A comparison of the first two hours of the height component output by OMNI and TRIMMBP using the Trimble day 341 data.

It is quite obvious that the initialization here was bad. Both the OMNI and the TRIMMBP solutions began with the same height, but they soon diverged before stabilizing with about a two meter offset between them.

3. Satellite Geometry

During each session, as satellites rose and set, the geometry of the GPS configuration changed. Since the receivers were tracking more than four satellites most of the time, the PDOP values were generally low. Figure 33 shows the change in PDOP with time for the period between 07:00 and 10:30 UTC, on day 341, when the reference trajectory overlaps with the GPS trajectories. The satellite geometry should be similar on the previous day, with about a four minute offset.

The PDOP values are recorded in the data, at each epoch, along with the distance measurements. A program was used to extract the times for each PDOP value. Another program was written to select points in the reference trajectory with a desired PDOP value. Thus it was possible to difference each GPS trajectory with reference points at times when the PDOP was a fixed, predetermined value. The baselines between the fixed reference station, Beach - Lab, and the ship antennas were short. Thus the PDOP values of the fixed station were used as an indicator of the overall satellite - receiver geometry. Table 15 presents a statistical sunmary of the differences, for the various PDOP values recorded.

Table 15. PDOP FACTOR COMPARISON: Differences in the ϕ, λ and h components between the GPS and the reference trajectories for various PDOP values.

DATA FILE	PDOP	NUMBER OF POINTS	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*
ASHTECH DAY 341	All	8500	-42 ± 64	28 ± 57	39 ± 53
	2	4580	-35 ± 62	45 ± 49	57 ± 48
	3	2810	-49 ± 65	18 ± 57	23 ± 49
	4	1240	-57 ± 64	-16 ± 54	5 ± 48
TRIMBLE DAY 341	All	8400	-202 ± 131	139 ± 128	-18 ± 310
	2	4700	-133 ± 70	195 ± 54	181 ± 70
	3	2940	-256 ± 114	112 ± 122	-186 ± 272
	4	730	-437 ± 127	-119 ± 144	-640 ± 166

As discussed in chapter II satellite geometry has an important effect on the accuracy achieved. The measurement accuracy is fixed and is determined by the equipment used. The positioning accuracy is a factor of the PDOP, $\sigma_x = PDOP \cdot \sigma_0$. This would suggest a deterioration of the positioning accuracy as the PDOP increases.

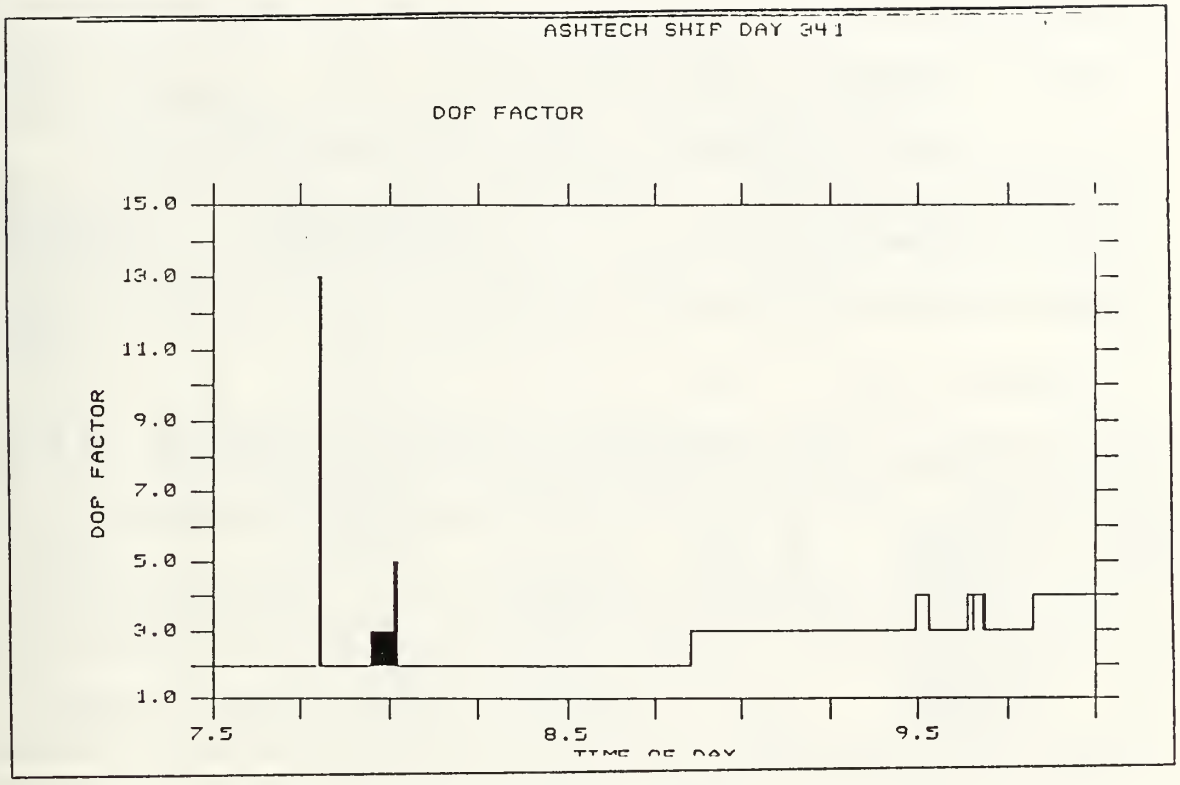

Figure 33. PDOP as a Function of Time - Day 341

The PDOP changed only slightly during the survey, with values between two and four for the most part. The values presented in table 15 show no significant changes in the accuracy as the PDOP increased.

4. Broadcast or Precise Ephemeris

The Ashtech and Trimble programs allow the user to choose the desired ephemeris file for creating the undifferenced phase measurements file. A comparison of the resulting trajectories, using the Broadcast and the Precise Ephemeris files, each differenced from the reference trajectory, is presented in table 16. The data used is the Ashtech day 341 data and the Trimble days 340 and 341 data, processed with the Trimble software.

Table 17 shows the differences between the trajectories for each receiver, once processed with the Broadcast Ephemeris and once with the Precise Ephemeris. The same

Table 16. **EPHEMERIS FILE COMPARISON:** Results of processing using Broadcast Precise Ephemeris files - average distance and standard deviation, in centimeters, fi the reference trajectory in the ϕ, λ and h components.

DATA / PROGRAM	DAY	Δ	BROADCAST EPHEMERIS	PRECISE EPHEMERIS	NO OF POI
ASHTECH / GPPS	341	$\Delta\phi$	2 ± 66	3 ± 66	850
		$\Delta\lambda$	59 ± 50	60 ± 53	
		Δh	111 ± 47	114 ± 47	
TRIMBLE / TRIMMBP	340	$\Delta\phi$	-164 ± 89	-197 ± 120	530
		$\Delta\lambda$	210 ± 65	196 ± 73	
		Δh	107 ± 182	29 ± 255	
TRIMBLE / TRIMMBP	341	$\Delta\phi$	-202 ± 131	-211 ± 142	840
		$\Delta\lambda$	139 ± 128	128 ± 146	
		Δh	-18 ± 310	-38 ± 333	

data as in the above table was used, only this time the full overlap from initialization was used, increasing the number of points used accordingly.

Table 17. **PRECISE - BROADCAST EPHEMERIS:** The trajectory files processed using Prec Ephemerises compared to the same data processed using Broadcast Ephemerises - d ferences in centimeters in the ϕ, λ and h components.

DATA	DAY	SOFTWARE	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*	NO. O POINT
ASHTECH	341	ASHTECH	1 ± 1.5	-2 ± 1.5	5 ± 4	15850
TRIMBLE	340	TRIMBLE	0.1 ± 0.7	-0.1 ± 0.5	0.6 ± 1.1	13900
	341		0 ± 2.8	-2.4 ± 4.5	5 ± 16	15580

Two receivers, on a short baseline, essentially see the satellites at the same angle. Any error in the satellite position, in any direction, will be observed by both receivers almost identically. The double differencing procedure should eliminate this effect from

the measurements. The values in table 17 are once again small, in accordance with our expectations.

5. Data Interval

All data were recorded at a one second interval. However all three programs, Ashtech, Trimble and NGS - OMNI, allow the user to determine the data interval for processing. The Ashtech and Trimble day 341 data were processed with a ten second interval to compare to the one second outputs. Table 18 shows the results of this comparison.

Table 18. RECORDING INTERVAL COMPARISON - DAY 341 DATA: Results of processing using a one second and a ten second interval - difference between output trajectories and reference in centimeters in the ϕ, λ and h components.

DATA / PROGRAM	Δ	ONE SEC. INTERVAL	NO. OF POINTS	TEN SEC. INTERVAL	NO. OF POINTS
ASHTECH / GPPS	$\Delta\phi$	2 ± 67	8500	2 ± 49	900
	$\Delta\lambda$	59 ± 50		59 ± 35	
	Δh	111 ± 47		111 ± 27	
TRIMBLE / TRIMMBP	$\Delta\phi$	-202 ± 131	8400	-209 ± 130	900
	$\Delta\lambda$	139 ± 138		131 ± 140	
	Δh	-18 ± 310		-34 ± 321	

Table 19. ONE SECOND - TEN SECOND INTERVAL DAY 341: The differences between one second interval trajectories and ten second interval trajectories, in centimeters, in the ϕ, λ and h components.

DATA	NO. OF POINTS	$\Delta\phi$ *cm*	$\Delta\lambda$ *cm*	Δh *cm*
ASHTECH	900	-5 ± 33	-7 ± 35	2 ± 30
TRIMBLE	900	-3 ± 8	1 ± 8	-7 ± 17

Table 19 shows the differences between the one second and the ten second interval trajectories. The data rate used in the processing should not have any effect on the results. Table 19 verifies this for the data examined, although the Ashtech solutions

show a larger standard deviation between the solutions. The noise in the solution is slightly reduced with a ten second interval used. The receivers used can record data at various rates, according to the user's input. The only consideration for selecting a certain data rate, either in the processing or in the data acquisition, should be computer storage space. One might argue that recording data at an infrequent rate will reduce the chances of cycle slips. This may be so, but since cycle slips are random phenomena, it is impossible to avoid them simply by selecting a low data recording rate. The ability to repair cycle slips depends on the time interval over which they occur. The longer the time lapse, the harder it is to interpolate accurately over the cycle slip. This suggests that it may be better to use a more frequent data recording interval.

6. Multipath

Multipath, especially on a moving platform, is a difficult parameter to assess. It cannot be determined directly by observation. However, correlation of errors (or offsets from the reference trajectory) with the direction in which the ship was moving, could indicate a multipath effect. Figure 34 shows the changes in the ship's heading as a function of time, for day 341.

The three revolutions of the ship around the pre-determined track are evident in the plots as are the times when the ship changed course. The plots were examined in conjunction with the error plots in each direction to try to find some correlation. No strong evidence to indicate that multipath was responsible for any of the trends or anomalies mentioned before, was found. Careful planning of the antenna array on the ship's mast, as well as relatively calm seas, appear to have afforded a low multipath environment during the experiment.

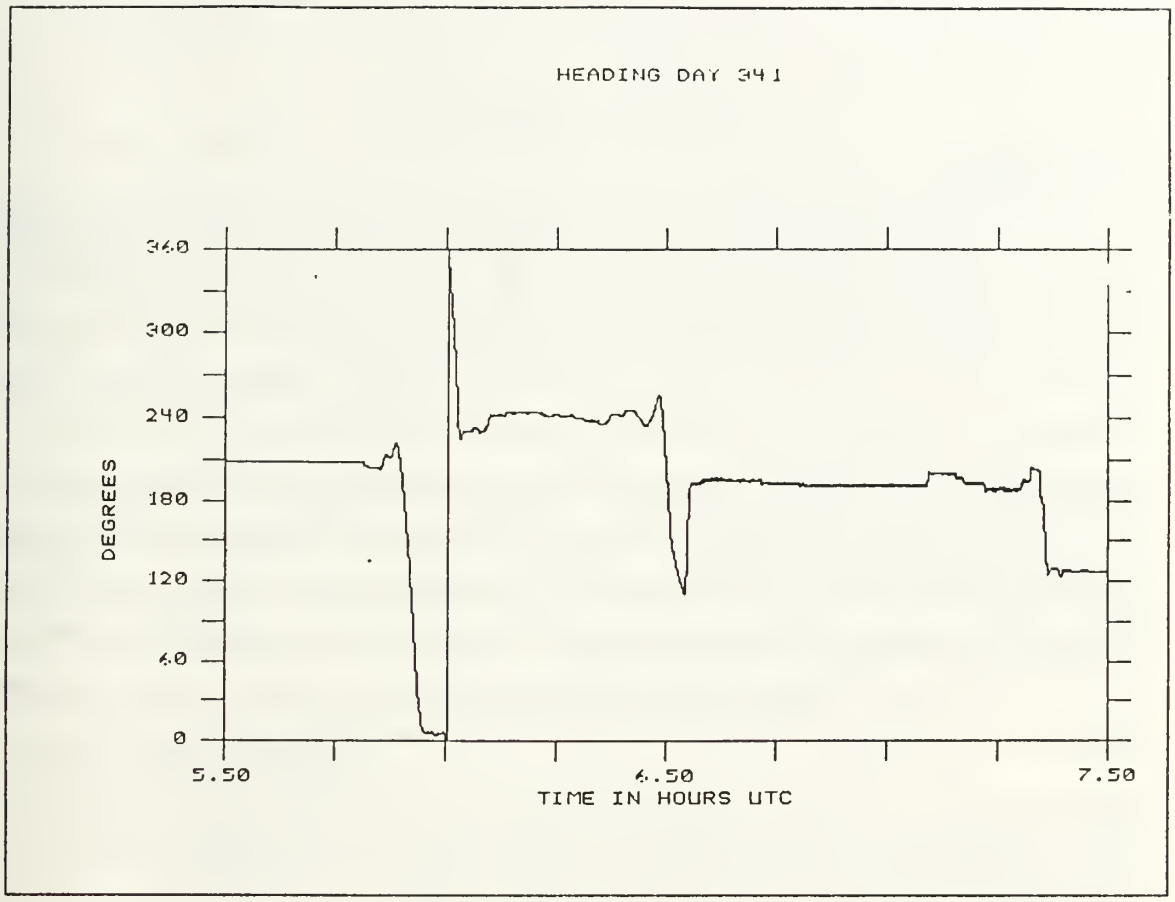

Figure 34. Ship's change in heading (Day 341)

VII. CONCLUSIONS

A. GENERAL

The objectives of this study were to determine the accuracy feasible in a shipborne kinematic GPS survey and the effects of certain factors on the accuracy achieved. A major factor which affected the study were the numerous problems encountered in getting the various programs to work. The example of the seven missing data points in the Ashtech day 340 data base, and the fact that the processing programs could not accommodate this gap, indicate a serious limitation in the software. The Trimble program uses a complex input file which calls various subroutines. While this structured processing technique has some advantages, it also complicates matters when simple changes are desired. An example of this is the option to use or ignore the tropospheric correction. In the Ashtech and NGS - OMNI programs, this option is easily chosen. The many attempts to exclude the tropospheric correction from the Trimble program, were all unsuccessful. OMNI was used with both the Ashtech and Trimble data. However, there were some difficulties in using OMNI with the Ashtech data, mainly due to format incompatibilities. These were eventually solved by NGS.

The first conclusions from this study pertain to this post - processing software comparison. It is imperative to point out that kinematic GPS surveying is not yet fully developed. Most companies, and users such as NGS, are continuing to develop and improve techniques to overcome the many idiosyncracies of kinematic GPS. Hopefully, future studies along the lines of this one, will not encounter the same problems. However the following comments on the various receivers and programs are appropriate at present.

The Ashtech receivers are easy to use, demand very little user - receiver interaction and are very versatile. The processing software, while easy to use, has serious limitations and is not particularly robust. However, when no serious problems, such as cycle slips, occur, the output is of high quality. The solution produced appears to be within a few centimeters, or tens of centimeters of the " truth ". Certainly it is within the noise level of the reference trajectory.

The Trimble receivers are also relatively easy to use. They demand more user-receiver interaction than the Ashtech, but certainly nothing complicated. The Trimble software, while seemingly more robust than the Ashtech, is also more complicated to use. The results indicate a strong sensitivity to the initialization. When a position, about two meters off the actual position, was used for the ship antenna at initialization, the solution appeared good for about two and a half hours, but rapidly deteriorated thereafter. When the correct position was used, the solution remained good throughout the session. It appears that changing satellite geometry and the setting and rising of satellites, caused the solution to deteriorate.

The NGS software has many advantages. It is general and can be used with various receivers' data. It enables the user to easily choose different options from a menu. It produces many plots in the interim stages which assist the user in finding problems which, when identified, can be fixed with certain algorithms applied. This however has the disadvantage of requiring a large amount of computer memory.

B. SOLUTION ACCURACY

It is evident from the results that highly accurate trajectories are feasible with kinematic GPS techniques. The Ashtech plots in particular shows signs of producing an output even more accurate than the reference trajectory itself. The results of the differencing procedure certainly fall within the noise level (about 60 cm) of the laser system.

The Ashtech data, processed with OMNI, seems to be of the same order of accuracy as the output obtained from GPPS. However, the limitation in OMNI, whereby ephemeris data for a certain satellite is necessary before the satellite is used, seriously constrains the program. The fact that neither software program could overcome the seven second gap in the Ashtech data, also has serious consequences, since cycle slips are to be expected in a shipborne survey.

The Trimble trajectory produced is of a similar accuracy to the Ashtech. However, the first processing attempts, with a baseline off by about two meters, caused the solution to deviate drastically after a period of time. Once this mistake was rectified and a correct baseline input used, the solution improved and remained steady throughout the session, giving comparible results to the Ashtech.

The trend in the height, when the bad baseline input was used, shows signs of being correlated to the satellites. This is due to the fact that it appears on both days, with a

four minute offset in time. It does not show any correlation with the ship's heading, PDOP or software used.

C. SUMMARY OF CONCLUSIONS

The main factors affecting the accuracy of shipborne GPS surveys are the initialization and the receiver and software package used. The other factors examined had little effect on the results. Table 20 provides a concise overview of the various program's advantages and limitations.

Table 20. SOFTWARE ATTRIBUTES

Attribute	Feature	GPPS	TRIMMBP	OMNI
Accuracy	Horiz. Bias	small (centimeters)	small (centimeters)	small (centimeters
	Vertical Bias	one meter	one meter	two meters from r erence
	Trends	none	none	none
	Anoma-lies	height begins 1 m below initialization height	height bias of about 1m	none
	Noise	low	low	low
Robustness	Cycle slip fixing	some capability	some capability	good capability
	Data gaps	cannot overcome	not tested	cannot overcome
Ease of Use	Option selection	easily applied	complex	easily applied
	Data Storage Space	relatively small	large	very large
Input	Data formats	Ashtech only	Trimble only	many receivers
	Ephemeris	Broadcast and Precise	Broadcast and Precise	Broadcast and Prec

APPENDIX A. GLOSSARY

1. ALMANAC - Data transmitted by a GPS satellite which includes orbit information on all the satellites, clock correction and atmospheric delay parameters. These data are used to facilitate rapid SV acquisition. The orbit information is a subset of the ephemeris data with reduced accuracy.

2. AMBIGUITY - The unknown integer number of cycles of the reconstructed carrier phase contained in an unbroken set of measurements from a single satellite pass at a single receiver.

3. BASELINE - The three - dimensional vector distance between a pair of stations for which simultaneous GPS data has been collected and processed with differential techniques.

4. C/A CODE - The Coarse/Acqasition (or Clear/Acquasition) code modulated onto the GPS L1 signal. This code is a sequence of 1023 pseudorandom binary biphase modulations on the GPS carrier at a chipping rate of 1.023 MHz, thus having a code repitition period of one millisecond. This code was selected to provide good acquisition properties.

5. CARRIER - A radio wave having at least one characteristic (such as frequency, amplitude, phase) which may be varied from a known reference value by modulation.

6. CARRIER BEAT PHASE - The phase of the signal which remains when the incoming Doppler - shifted satellite carrier signal is beat (the difference frequency signal is generated) with the nominally constant reference frequency generated in the receiver.

7. CARRIER FREQUENCY - The frequency of the unmodulated fundamental output of a radio transmitter. The GPS L1 carrier frequency is 1575.42 MHz.

8. DIFFERENTIAL (RELATIVE) POSITIONING - Determination of relative coordinates of two or more receivers which are simultaneously tracking the same satellites. Dynamic Differential positioning is a real - time calibration technique achieved by sending corrections to the roving user from one or more monitor stations.

9. DIFFERENCIAL PROCESSING - GPS measurements can be differenced between receivers, satellites and epochs. Although many combinations are possible, the present convention for differential processing of GPS phase measurements is to take differences between receivers (single differences), then between satellites (double differences), then between epochs (triple differences).

   A single - difference measurement between receivers is the instantaneous difference in phase of the signal from the the same satellite, measured by two receivers simultaneously.

   A double - difference measurement is obtained by differencing the single difference for one satellite with respect to the corresponding single difference for a chosen reference satellite.

   A triple - difference measurement is the difference between a double difference at one epoch of time and the same double difference at a previous epoch of time.

10. DILUTION OF PRECISION (DOP) - A description of the purely geometrical contribution to the uncertainty in a position fix, given by the expression DOP = SQRT TRACE (AA) where AA is the design matrix for the instantaneous position solution (dependant on satellite - receiver geometry). The DOP factor depends on the parameters of the position - fix solution. Standard terms for the GPS application are :

    GDOP - Geometric (three position coordinates plus clock offset in the solution).

    PDOP - Position (three coordinates only).

    HDOP - Horizontal (two horizontal coordinates only).

    VDOP - Vertical (height parameter only).

    TDOP - Time (clock offset parameter only).

    RDOP - Relative (normalized to 60 seconds).

11. DoD - Department of Defense.

12. DOPPLER SHIFT - The apparent change in frequency of a received signal due to the rate of change of the range between the transmitter and the receiver. See reconstructed carrier phase.

13. EARTH - CENTERED - EARTH - FIXED (ECEF) - Cartesian coordinate system where the X direction is the intersection of the prime meridian (Greenwich) with the equator. The vectors rotate with the Earth (hence Earth fixed). Z is in the direction of the spin axis and Y completes a right handed orthogonal coordinate system.

14. ELEVATION - Height above mean sea level. Vertical distance above the geoid.

15. ELEVATION ANGLE - The angle, above the horizon, at which the satellite is seen by the receiver.

16. ELEVATION MASK ANGLE - That angle below which it is recommended that satellites not be tracked (or if tracked, not used in processing). The common elevation mask angles used are 10 or 15 degrees to avoid interference problems caused by tall buildings, trees, terrain etc. and multipath errors.

17. EPHEMERIS - A list of (accurate) positions or locations of a celestial object as a function of time. Available in GPS terms as " Broadcast Ephemeris " or as postprocessed " Precise Ephemeris. "

18. EPOCH - Measurement interval or data frequency, as in making observations every 15 seconds.

19. EXTRATERRESTRIAL - existing outside the earth or it's atmosphere.

20. GEOCENTER - The center of the earth.

21. GEOID - The particular equipotential surface which coincides with mean sea level, and which may be imagined to extend through the continents. This surface is everywhere perpendicular to the force of gravity.

22. GEODETIC HEIGHT - Also called ellipsoidal height, this is the height above the reference ellipsoid, measured along the ellipsoidal outer normal through the point in question.

23. INTEGER BIAS TERMS - The receiver counts the radiowaves from the satellite, as they pass the antenna, to a high degree of accuracy. However, it has no infor-

mation on the number of waves to the satellite at the time it started counting. This unknown number of wavelengths between the satellite and the antenna is the integer bias term.

24. INTEGTRATED DOPPLER - A measurement of Doppler shift frequency or phase over time.

25. IONOSPHERIC DELAY - A wave propagating through the ionosphere (which is a nonhomogenous [in space and time] and dispersive medium) experiences delay. Phase delay depends on electron content and affects carrier signals. Group delay depends on dispersion in the ionosphere as well, and affects signal modulation (codes). The phase and group delay are of the same magnitude but opposite in sign.

26. KINEMATIC SURVEYING - A form of continuous differential carrier - phase surveying requiring only short periods of data observations at each site. Continuous Kinematic surveying refers to determining the trajectory of the receiver antenna which is in continuous motion (for example mounted on a ship/plane/land vehicle).

27. L1 - The primary L - band signal transmitted by each NAVSTAR satellite at 1575.42 MHz. (Usually the frequency chosen for use with single frequency receivers).

28. L2 - The secondary L -band signal transmitted by each NAVSTAR satellite at 1227.60 MHz. (Usually used only by dual frequency receivers).

29. L BAND - The radio frequency band extending from 390 MHz to (nominally) 1550 MHz.

30. MICROSTRIP ANTENNA - A two dimensional, flat, precisely cut piece of metal foil glued to a substrate (groundplane).

31. MONITOR STATION - Worldwide group of stations used in the GPS control segment to monitor satellite clock and orbital parameters. Data collected here are linked to a Master Station where corrections are calculated and controlled. These data are uploaded to each satellite at least once per day from an Upload Station.

32. MULTICHANNEL RECEIVER - A receiver containing many independent channels for satellite tracking.

33. MULTIPATH - Interference similar to " ghosts " on a television screen which occurs when GPS signals arrive at an antenna having traversed different paths. The signal traversing the longer path will yield a larger pseudo range estimate and increase the error. Multiple paths may arise from reflections from structures near the antenna.

34. MULTIPATH ERROR - A positioning error resulting from interference between radiowaves which have traveled between the transmitter and the receiver by two paths of different electrical lengths.

35. NAVSTAR - The name given to GPS satellites, built by Rockwell International, which is an acronym formed from NAVigation System with Timing and Ranging.

36. NGS - National Geodetic Survey.

37. OBSERVING SESSION - The period of time over which GPS data is collected simultaneously by two or more receivers.

38. P - CODE - The protected or precise code used on both L1 and L2 GPS beacons.This code will be made available by the DoD only to authorized users. The P - code is a very long (about 10**14 bits) sequence of pseudorandom binary biphase modulations on the GPS carrier at a chipping rate of 10.23 MHz which does not repeat itself for about 38 weeks. Each satellite uses a one - week segment of this code which is unique to each GPS satellite and is reset each week.

39. PHASE LOCK - The technique whereby the phase of an oscillator signal is made to follow exactly the phase of a reference signal by first comparing the phases of the two signals, then using the resulting phase difference signal to adjust the reference oscillator frequency to eliminate phase difference when the two signals are next compared.

40. PRECISE POSITIONING SERVICE - The highest level of military dynamic positioning accuracy that will be provided by GPS, based on the dual - frequency P - code and having high anti - jam and anti - spoof qualities.

41. PRN - Pseudorandom noise, a sequence of digital 1's and 0's which appears to be randomly distributed like noise, but which can be exactly reproduced. The important property of PRN codes is that they have a low autocorrelation value for all delays or lags except when they are exactly coincident. Each NAVSTAR satellite has its own unique C/A and P pseudorandom - noise codes.

42. PSEUDORANGE - A measure of the apparent propagation time from the satellite to the receiver antenna, expressed as a distance. Pseudorange is obtained by multiplying the apparent signal - propagation time by the speed of light. Pseudorange differs from the actual range by the amount that the satellite and user clocks are offset, by propagation delays and other errors.

43. RANGE RATE - The rate of change of range between the satellite and receiver. The range to a satellite changes due to satellite and observer motions. Range rate is determined by measuring the Doppler shift of the satellite beacon carrier.

44. RECONSTRUCTED CARRIER PHASE - The difference between the phase of the incoming Doppler shifted GPS carrier and the phase of a nominally constant reference frequency generated in the receiver. For Static positioning the reconstructed carrier phase is sampled at epochs determined by a clock in the receiver.

    The reconstructed carrier phase changes according to the continuously integrated Doppler shift of the incoming signal, biased by the integral of the frequency offset between the satellite and the receiver reference oscillators.

    The reconstructed carrier phase can be related to the satellite - to - receiver range once the initial range (or phase ambiguity) has been determined. A change in the satellite - to - receiver range of one wavelength of the GPS carrier (19 cm for L1) will result in a one - cycle change in the phase of the reconstructed carrier.

45. ROVER - A moving receiver in the kinematic mode of GPS.

46. RTCM - Radio Technical Commision for Maritime Services. Commision set up to define a differential data link to relay GPS correction messages from a monitor station to a field user. RTCM SC - 104 recommendations define the correction message format and 16 different correction message types.

47. SELECTIVE AVAILABILITY (SA) - A DoD program to control the accuracy of pseudorange measurements, whereby the user receives a false pseudorange which is in error by a controlled amount. Differential GPS techniques can reduce these effects for local applications.

48. STATIC POSITIONING - Positioning applications in which the positions of static or near static points are determined.

49. SV - Satellite vehicle or space vehicle.

50. TOW - Time of week, in seconds, from midnight Sunday UTC.

51. TROPOSHERIC CORRECTION - The correction applied to the measurement to account for tropospheric delay. This value is obtained from one of various models used which input the atmospheric parameters - temperature, pressure and relative humidity - and output the correction factor.

52. UNIVERSAL TIME (UT) - Local solar mean time at Greenwich Meridian.

53. UNIVERSAL TIME COORDINATED (UTC) - Uniform atomic time system kept very close to UT corrected for polar motion and seasonal variations in the earth's rotation rate. Maintained by the U.S. Naval Observatory and corrected with leap seconds when necessary.

54. USER RANGE ACCURACY (URA) - The contribution to the range - measurement error from an individual error source (apparent clock and ephemeris prediction accuracies), converted into range units, assuming that the error source is uncorrelated with all other error sources.

55. WORLD GEODETIC SYSTEM - 84 (WGS - 84) - The mathematical ellipsoid used by GPS since January 1987.

56. WIDELANE - A linear combination of L1 and L2 observations used to partially remove ionospheric errors.

APPENDIX B. ADJUSTMENT COMPUTATIONS AND LINEARIZATION TECHNIQUES

We have described the measurements and the observation equations for the various models. In order to determine the required parameters, receiver coordinates at each instant, we need to relate the measurements to the observation equations. This is done in a least squares process. A general overview of the least squares technique is given here.

The least squares non - linear mathematical model is of the form :

$$L^a = F(X^a)$$

where L^a are the theoretical, calculated values of the observed quantaties and X is a vector of the theoretical values of the parameters (coordinates). The superscript " a " pertains to the adjusted inputs used in calculating the observed quantities. Estimating the parameters, or using adjusted values of the parameters, results in the equation :

$$\hat{L}^a = F(\hat{X}^a)$$

where the $\hat{X}$ indicates estimated or adjusted values. The observed values, including the errors, are given by the equation :

$$L^b - \varepsilon = F(X^a)$$

where ε is the " true error " and L^b is the actual observed values of the observable (time, phase angle ....). Comparing the observed values to the theoretical, calculated values produces : $L^b - \varepsilon = F(X^a)$ but this calculated value can be expressed as a Taylor series expansion around an estimated value - X^0 resulting in :

$$L^b - \varepsilon = F(X^a) = F(X^0) + \frac{\partial F}{\partial X^a} \Big|_{x^a = x^0} (X^a - X^0) + \ldots$$

In this case we are linearizing by truncating the Taylor series expansion at the first derivitive term. $\frac{\partial F}{\partial X^a} \Big|_{x^a = x^0} = A$ which is called the design matrix. If we have n observations (measurements) and u parameters to solve for (coordinates, integer ambiguity ...) the resulting design matrix will be of the size u x n.

Rearranging the previous equation and substituting in A for the partial derivitive, we get

$$-\varepsilon = AX + F(X^0) - L^b \quad where \quad X = X^a - X^0; \qquad F(X^0) = L^0$$

Using the estimated parameter the error becomes a residual " V " and the above equation takes the form of :

$$V = A\hat{X} + F(X^0) - L^b \quad where \quad \hat{X} = \hat{X}^a - X^0; \qquad L^0 - L^b = L$$

The least squares observation equation now becomes $V = A\hat{X} + L$. What we want is a minimum variance solution. Using a weight matrix, P, we wish to minimize V^TPV which is

$$\begin{aligned} V^TPV &= (A\hat{X} + L)^TP(A\hat{X} + L) \\ &= \hat{X}^TA^TPA\hat{X} + \hat{X}^TA^TPL + L^TPA\hat{X} + L^TPL \end{aligned}$$

Since, ${}_1\hat{X}^T_{uu}A^T_{nn}P_{nn}L_1 = {}_1L^T_{nn}P_{nn}A_{uu}\hat{X}_1$ we need to minimize

$$V^TPV = \hat{X}A^TPA\hat{X} + 2L^TPA\hat{X} + L^TPL$$

We minimize this by partially differentiating with respect to $\hat{X}$ and equating to zero. The result is :

$$\hat{X} = -(A^TPA)^{-1}A^TPL.$$

Recalling that $\hat{X} = \hat{X}^a - X^0$ our adjusted values are obtained by adding $\hat{X}$ to our initial estimated values X^0. Iterating until $\hat{X}$ becomes less than a preset tolerance, produces the least squares solution for determining the parameters.

The variance - covariance matrix of adjusted parameters is a u x u matrix whose diagonal elements contain the variances of the different parameters. The variance - covariance matrix of X is defined by

$$\Sigma_x = E\left[[X - E(X)][X - E(X)]^T\right]$$

where E stands for the expected value of the parameter. This reduces to the matrix

$$\Sigma_x = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \cdots\cdots\sigma_{x_1x_n} \\ \sigma_{x_2x_1} & \sigma_{x_2}^2 & \cdots\cdots\sigma_{x_2x_n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \sigma_{x_nx_1} & \sigma_{x_nx_2} & \cdots\cdots\sigma_{x_n}^2 \end{bmatrix}$$

APPENDIX C. COMPUTER REQUIREMENTS FOR THE MBPPE.

This appendix is a concise summary of the GPS data files collected during the experiment, the processed trajectory files and the reformatted files used in the differencing programs. The main objective of this appendix is to provide pertinent information, particularly of file size and processing time, for each of the data sets used.

The computer used for most of the processing was a 25 MHz 386 with 120 MB hard drive, 4 MB RAM and an 80387 math co - processor. Table 21 shows the Ashtech data files collected during the two days of kinematic surveying. At the time the data was collected the Ashtech software had an expanded data format, where channels not being used in the receiver put out zeros for data. In the newer software this inefficient data storage system has been changed and the files are more condensed. A program enables the user to convert expanded format data to compressed format data for use with the new version software.

Table 21. ASHTECH DOWNLOADED DATA FILES

Day	Raw Data File	Expanded	Compacted
340	BDOP1390.340	16 MB	8 MB
	BSHIP390.340		
341	BDOP1490.341	16 MB	8 MB
	BSHIP490.341		

The ephemeris data (" E ") files are about 6.5 KB each and the site (" S ") files are about 0.5 KB each. The " U " files created for processing are about 1.4 times the size of the original raw data files. This results in two files of over 12 MB created. However, once created, the original raw data is no longer needed for processing. Two " U " files for any day were on the order of 25 MB together. The full output file after processing five and three quarter hours of data, was over 5 MB in size. Reformatting to contain only times, latitude, longitude and height, condensed the trajectory file to about 1 MB.

Table 22 shows the Trimble files downloaded from the PC where the data was logged. The data - logger stores data in a compressed mode and expands it in the downloading process.

Table 22. TRIMBLE DOWN-LOADED DATA FILES

Day	Raw data file	Size
340	BLAB3400.DAT	10.81 MB
340	SHIP3400.DAT	10.26 MB
341	BLAB3410.DAT	10.0 MB
341	SHIP3410.DAT	9.88 MB

The ephemeris files (.EPH) are 10.75 KB each, the message files (.MES) are about 1.6 KB each and the ionosphere files (.ION) are 115 bytes each.

The scratch file created by the Trimble software combines the data files into undifferenced data for processing. This file, which exists together with the raw data files, is about two times the size of the original raw data files combined. This resulted in scratch files on the order of 23 MB being created for four and a quarter hours of processing data. The maximum number of points that can be processed by the Trimble software is 16,000 - just over four and a quarter hours at a 1 second recording interval. The output files, resulting from the processing and containing dx, dy and dz offsets from the reference station, are over 1.25 MB in size. Converting these files to the reference trajectory format with times, latitude, longitude and height, creates files of about 8 MB.

The reference trajectory files are about 400 KB each. The output of the differencing program which compared the GPS trajectories to the reference trajectories, was about 500 KB in each case.

LIST OF REFERENCES

1. Schwarz, Cannon and Wong, Manuscripta Geodetica (West Germany) 1989 , volume 14 pages 345 - 353. *A Comparison of Kinematic Models for the Determination of Position and Velocity Along a Trajectory.*

2. Mader G.L., Journal of Geophysical Research, volume 94 no. 88, August 1989. *Verification of Airborne Positioning Using GPS Carrier Phase Measurements.*

3. Heiskanen W.A. and Moritz H., *Physical Geodesy,* Institute of Physical Geodesy, Technical University, Graz, Austria 1987.

4. Leick A., *GPS Satellite Surveying,* Department of Surveying Engineering, University of Maine, 1990.

5. *GPS - ICD - 200,* Rockwell International Corporation, Satellite Systems Division, Navstar GPS Space Segment / Navigation User Interface, Revision B, 30 November 1987.

6. Wells D., *Guide to GPS Positioning,* Canadian GPS Associates, May 1987.

7. Captain K.J. Schnebbele, Internal Memorandum *Geodetic Survey Report for the Monterey Bay Precise Positioning Experiment,*Dept. of Oceanography, Naval Postgraduate School, CA, January 8, 1991.

8. Mader G.L., Schenewerk M.S. and Chin M.M., *OMNI 2.00 User's Guide,* National Geodetic Survey, Rockville, Maryland, May 1, 1991.

9. Clynch J.R. et al, Proceedings of the ION GPS - 91 Technical Program, (to be published), *Monterey Bay Precise Positioning Experiment, Comparisons of GPS Receiver Solutions Under Dynamic Conditions.*

10. Hamming, R.W., *Digital Filters* Naval Postgraduate School, Ca, 1983.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 52 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Applied Research Laboratories University of Texas at Austin P.O. Box 8029 Austin TX 78713 - 8029 Attention : Mr. D. Coco Mr. M. Leach Mr. B. Tolman	1
4.	Ashtech Inc. 390 Potrero Ave. Sunnyvale CA 94086 Attention : Mr. M. Evers Mr. P. Heineman	2
5.	Defense Mapping Agency System Center 8613 Lee Highway Fairfax VA 22031 -2138 Attention : Mr. R. Ziegler Mr. S. Malys Dr. M. Kumar	3
6.	Defense Mapping Agency Headquarters 8613 Lee Highway Fairfax VA 22031 - 2138 Attention : Mr. J. Slater Mr. T. Meyer	2
7.	Defense Mapping Agency H.T.C. Commander Geodetic Survey Group Attention (GGPP)	2

P.O. Box 9617
F.E. Warren AFB
WY 92005 - 6300
Attention :
Mr. H. Heurmam
Mr. J. Rees

8. Israeli Naval Attache — 4
c/o Israeli Embassy
3514 International Drive N.W.
Washington D.C. 20008
Attention LCDR. B. Grinker
Naval Hydrographic Office - Tel Aviv

9. Monterey Bay Acquarium Research Institute — 1
160 Central Ave.
Pacific Grove
CA 93950
Attention Mr. A. Pierce

10. National Geodetic Survey — 2
N/CG 114
11400 Rockville Pike
Rockville
MD 20852
Attention :
Dr. G.Mader
Ms. M. Chin

11. Naval Air Development Center — 1
Warminster
PA 18974 - 5000
Attention Mr. V. Dicaistofaro

12. Naval Air Warfare Center - Weapons Division — 2
Pt. Mogo
CA 93042 - 5000
Attention :
Mr. B. Cohenour
Mr. R. Smith

13. Naval Oceanographic Center — 1
Stennis Space Center
MS 39922 - 5001
Attention Mr. P. Taylor (Code HS)

14. Naval Postgraduate School — 5
Department of Oceanography
Monterey
CA 93943
Attention :
Dr. C. Collins

Dr. J. Clynch
Dr. H. Titus

15. Naval Surface Weapons Center - Dahlgren Laboratory — 3
Dahlgren
VA 22448
Attention :
Mr. R. Hill (K10)
Mr. A. Evans (K10)
Mr. B. Herman (K10)

16. Navy Space Systems Activity — 1
Code (80)
P.O. Box 92960
Los Angeles Air Force Station
Los Angeles
CA 90009 - 2960
Attention CMDR J.K. Mc. Dermott

17. NOAA — 1
6001 Executive Blvd
Rockville
MD 20852
Attention Mr. J. Wallace

18. Trimble Navigation — 2
645 N. Mary Ave.
P.O. Box 3642
Sunnyvale
CA 94088 - 3642
Attention :
Mr. D. Young
Dr. R. Kalifus

19. Texas State Department of Public Transportation — 1
11th and Brazos St.
Austin
TX 78701
Attention Mr. F. Howard (D19)

20. USGS — 1
Menlo Park
CA 94025
Attention Mr. W. Prescott

Thesis
G8337 Grinker
c.1 Accuracy of shipborne
kinematic GPS surveying.

3 2768 00033274 6